## Statistical Models

246. Suppose the length of a MWF English class is a random variable $X \sim N(48, 2.25)$ minutes.

   (a) Find the 15th percentile.
   **Answer:** $invnorm(.15, 48, 2.25) = 45.67$

   (b) Find the probability that class lets out late ($X > 50$).
   **Answer:** $normcdf(50, E99, 48, 2.25) = .187$

   (c) Find the probability that class lets out late on at least 3 of the next 9 class periods.
   **Answer:** $1 - bicdf(9, .187, 2) = .228$

247. If individual blood glucose levels are $x \sim N(110, 30)$, then

   (a) What percentage of people have dangerously high levels (above 150)?
   **Answer:** $normcdf(150, E99, 110, 30) = .091$

   (b) Fill in the blank: one third of people have blood sugar levels between 100 and _____.
   **Answer:** draw a picture! $normcdf(-E99, 100, 110, 30) = .37$, so we need $invnorm(.37 + .333, 110, 30) = 126$

   (c) Find $Q_1$ and $Q_3$ for this distribution (the values that mark the middle 50%)
   **Answer:** $Q_1 = invnorm(.25, 110, 30) = 89.77$ and $Q_3 = invnorm(.75, 110, 30) = 130.23$

   (d) In a group of 75 people that ate a certain diet, only 3 of them had high (above 150) levels. Use the binomial distribution to find the probability that 3 or fewer would occur by luck.
   **Answer:** for each individual, $p = .091$, so we get $bicdf(75, .091, 3) = .081$

248. Suppose annual rainfall is $x \sim N(36, 8)$ inches. A year is considered "dry" if the rainfall is under 30. What is the probability of having more than 3 dry years in a decade?
**Answer:** For any single year, we get $p = normcdf(-E99, 30, 36, 8) = .227$. For getting more than 3 out of 10, we get $1 - bicdf(10, .227, 3) = .173$

249. Uncle Rico has a powerful arm. The distance he can throw a football is $x \sim N(45, 4)$ yards.

   (a) What is the probability that he throws it between 40 and 50 yards?
   **Answer:** $normcdf(40, 50, 45, 4) = .789$

   (b) What is the probability that he throws it at least 50 yards?
   **Answer:** $normcdf(50, E99, 45, 4) = .106$

   (c) What is the probability that he makes three consecutive throws of at least 50 yards?
   **Answer:** $(.106)^3 = .0012$

(d) What is the probability that he has at least three 50+ yard throws in twenty trys?
**Answer:** $1 - bicdf(20, .106, 2) = .357$

(e) What distance would a throw go if it is at the 95th percentile?
**Answer:** $invnorm(.95, 45, 4) = 51.6$ yards

(f) Fill in the blank: 50 percent of his throws are between 40 and _____ yards.
**Answer:** draw a picture! note the tail to the left of 40 has area $normcdf(-E99, 40, 45, 4) = .106$, so the right endpoint is $invnorm(.606, 45, 4) = 46.08$

250. A medical device uses a battery with lifespan distributed $x \sim N(300, 24)$ days. Out of 5000 such batteries,

(a) How many of them would be expected to last less than 250 days?
**Answer:** $normcdf(-E99, 250, 300, 24) \cdot 5000 = (.0186)(5000) = 93$

(b) Find the probability that at least 100 of the 5000 batteries will last less than 250 days.
**Answer:** $1 - bicdf(5000, .0186, 99) = .245$

251. Suppose a basketball team's point total in a game is approximated by $X \sim N(75, 11)$. Using a continuity correction,

(a) find the probability that they score at least 90 points.
**Answer:** $normcdf(89.5, E99, 75, 11) = .094$

(b) find their 80th percentile score
**Answer:** $invnorm(.8, 75, 11) = 84$

(c) find the probability that they score exactly 75 points.
**Answer:** $normcdf(74.5, 75.5, 75, 11) = .036$

252. The number in attendance at church Sunday morning is modeled by $x \sim N(460, 50)$, with continuity correction.

(a) Is the actual attendance discrete or continuous?
**Answer:** discrete, although the model is continuous, so we will need to round

(b) Find the probability of having at least 500 attendance.
**Answer:** $normcdf(499.5, E99, 460, 50) = .215$

(c) Find the probability of having between 450 and 490 in attendance.
**Answer:** $normcdf(449.5, 490.5, 460, 50) = .312$

(d) If only 400 attend, that corresponds to the _____ percentile.
**Answer:** $normcdf(-E99, 400.5, 460, 50) = .117$ so the 12th percentile

(e) Find the quartiles $Q_1$ and $Q_3$.
**Answer:** $Q_1 = invnorm(.25, 460, 50) = 426$ and $Q_3 = invnorm(.75, 460, 50) = 494$

253. Suppose $X \sim BI(100, 0.8)$.

(a) Find $\mu$ and $\sigma$.
**Answer:** $\mu = (100)(.8) = 80$ and $\sigma = \sqrt{(100)(.8)(.2)} = 4$

(b) Describe the shape of the distribution of $X$.
**Answer:** left skewed

(c) If you observed a sample of 64 values of $X$, then the sample mean $\overline{x}$ has mean $\mu_{\overline{x}} =$ _____ and standard deviation $\sigma_{\overline{x}} =$ _____.
**Answer:** $\mu_{\overline{x}} = 80$, $\sigma_{\overline{x}} = \sigma/\sqrt{64} = 4/8 = .5$

(d) Describe the shape of the distribution of $\overline{x}$.
**Answer:** nearly bell-shaped

254. Suppose that a civic arena seats 3000 people. Historically about 87% of ticketholders show up to a particular event (e.g. a concert or a game). If you sold 3400 general admission tickets,

   (a) How many ticketholders do you expect to attend the event? What is the standard deviation?
   **Answer:** $\mu = np = (3400)(.87) = 2958$ and $\sigma = \sqrt{np(1-p)} = 19.61$

   (b) Use bicdf to get the exact probability that there will be enough seats.
   **Answer:** $bicdf(3400, .87, 3000) = .986$

   (c) Use normcdf to approximate the probability that there will be enough seats. (do not worry about continuity correction)
   **Answer:** $normcdf(-E99, 3000, 2958, 19.61) = .984$

   (d) Fill in the blank: 75% of the time attendance will be fewer than _____
   **Answer:** $invnorm(.75, 2958, 19.61) = 2971$

255. Suppose a baseball player is truely a $p = .300$ hitter. Suppose he has 500 at-bats in a season. Let $x \sim BI(500, .3)$ be the number of hits he gets.

   (a) Find $\mu$.
   **Answer:** $\mu = (500)(.3) = 150$

   (b) Find $\sigma$.
   **Answer:** $\sigma = \sqrt{(500)(.3)(.7)} = 10.25$

   (c) Use a normal approximation to find the probability that he bats at least .320 (160 hits). Do not worry about continuity correction.
   **Answer:** $normcdf(160, E99, 150, 10.25) = .165$

256. Suppose $x \sim BI(75, p)$ and $\mu = 21$. Find $p$.
**Answer:** solve $np = \mu$ to get $p = .28$

257. Suppose $x \sim BI(n, .8)$ and $\sigma = 10$. Find $n$.
**Answer:** solve $\sqrt{np(1-p)} = \sigma$ to get $n = 625$

258. Suppose $x \sim BI(n, p)$ and $\mu = 32$ and $\sigma = 5.185$. Find $n$ and $p$.
**Answer:** you know $np = 32$, so $\sqrt{32(1-p)} = 5.185$. Solve to get $p = .16$. Then solve $n(.16) = 32$ to get $n = 200$.

259. If $X \sim N(90, 25)$ and $Y \sim BI(120, .75)$ are independent, find $P(X < 80 \text{ and } Y < 80)$.
**Answer:** $normcdf(-E99, 80, 90, 25) \times bicdf(120, .75, 79) = .00534$

260. According to the empirical rule, about 99.7% of observations from a normal distribution should be with 3 standard deviations of the mean, that is $|z| < 3$. Suppose you take $n = 7000$ measurements from a distribution that looks bell-shaped.

   (a) How many would you "expect" to be in the tails $|z| \geq 3$ ?
   **Answer:** $(.003)(7000) = 21$

   (b) Suppose you actually observed 85 such measurements. Do you think the distribution is normal?
   **Answer:** no, it appears to have fat tails since there are way more outliers than predicted by the normal dist.

261. Suppose the distribution of $X$ is normal with mean 100, and the z-score of $x = 172$ is three. In a sample of 7, what is the probability that a majority are over 95?
**Answer:** Since $\frac{172-100}{\sigma} = 100$, the standard deviation is 24. The probability of one observation being over 95 is $normcdf(95, E99, 100, 24) = .5825$, so the probability of a majority (at least 4) is $1 - bicdf(7, .5825, 3) = .676$

262. Suppose a case of chips contains 48 bags. If bag weights have a standard deviation of 1.5 oz, find the probability that a case weighs one pound more than usual. (assume independence and use the CLT)
**Answer:** by the CLT, the average bag's weight is $\overline{x} \sim N(\mu, 1.5/\sqrt{48})$ oz. For the case to be one pound over-weight, the average bag would have to be $16/48 = .333$ oz over-weight. So then $normcdf(.333, E99, 0, 1.5/\sqrt{48}) = .0618$

263. In a group of 15 mothers, 6 of them claimed that their child was in the top quintile of intelligence. What is the probability that in a random sample of 15 children, at least 6 of them would be above the 80th percentile?
**Answer:** the model is $X \sim BI(15, .2)$, so $P(X \geq 6) = 1 - bicdf(15, .2, 5) = .061$

264. In a sample of $n = 400$ individuals from a normal distribution,

   (a) What proportion would you expect to be in the tails: $|z| > 2$ ?
   **Answer:** $2 * normcdf(2, E99, 0, 1) = .0455$

   (b) How many individuals would that be?
   **Answer:** $(.0455)(400) \approx 18$

   (c) Find the probability that at least one individual has $|z| > 3$.
   **Answer:** $2 * normcdf(3, E00, 0, 1) = .00270$, then $1 - bicdf(400, .0027, 0) = .661$

## Sampling

265. Is it an observational study or a designed experiment?

   (a) A sociologist gives a survey about religious beliefs.
   **Answer:** designed experiment

   (b) A scientist tests whether sleep deprived mice do poorer in maze tests.
   **Answer:** designed experiment

   (c) data is collected about the lifespan of NFL players who have suffered concussions
   **Answer:** obs study

   (d) data is collected about whether rats that are given steroids have increased cancer risk
   **Answer:** designed experiment

   (e) a farmer tries different types of fertilizer on selected plots of land, and records the harvest yield of each
   **Answer:** designed experiment

   (f) data is collected about whether children from one-parent homes are more likely to commit suicide.
   **Answer:** obs study

   (g) a bored driver counts the number of men and women drivers that he passes, or pass him, on the interstate
   **Answer:** obs study

266. Is it a simple random sample (SRS), a stratified sample (SS), or a voluntary response sample (VRS)?

   (a) An urban charter school is opening and has 100 open spots, so a lottery is held for the 500 applicants. Each potential student has a ball in the hopper.
   **Answer:** SRS

   (b) Wal-mart does random drug tests by randomly selecting 10 full-time and 10 part-time employees.
   **Answer:** SS

   (c) Prize winners are selected by pulling names out of a (well mixed) hat.
   **Answer:** SRS

(d) ESPN.com asks viewers if they root for the AFC or the NFC in Super Bowls.
**Answer:** VRS

(e) The sociology department pays students to participate in an experiment.
**Answer:** VRS

(f) 5 students from each class (FR,SO,JR,SR) are randomly selected to fill out a cafeteria survey.
**Answer:** SS

(g) The census bureau randomly selects 50 county households to visit and administer an employment survey.
**Answer:** SRS

267. Would the sample be considered with or without replacement?

(a) Names are picked out of a hat for door prizes, and nobody can win more than once.
**Answer:** without

(b) A poker player is dealt 5 cards.
**Answer:** without

(c) A shooter attempts 10 free throws and records whether she made or missed each one.
**Answer:** with

(d) A scientist tags 15 bears for tracking.
**Answer:** without, no bear would be tagged twice

(e) There are 100 trees in a large open field. Three bolts of lightning strike, each one hitting a single tree.
**Answer:** with, lightning could strike the same tree twice

268. For each scenario, comment on potential flaws in the sampling procedure.

(a) A researcher polls Wal-mart shoppers about who they will vote for in the upcoming election.
**Answer:** Non-representative sample. Wal-mart shoppers tend to me more rural and lower-middle class. Also, many wal-mart shopper won't vote in the election.

(b) A student has cheated on two tests, and didn't get caught either time. He concludes that he will never get caught.
**Answer:** small sample size

(c) An online dating site asks users to report their salary.
**Answer:** self-reporting, tend to exaggerate

(d) An advocacy group surveys people with the question: "Do you agree with the majority of scientists that global warming is a serious man-made threat?"
**Answer:** leading/framing question

(e) The first ten students to arrive to class are asked if they did their homework.
**Answer:** selection bias, presumably the early students are more conscientious and may be more likely to do HW

## Inference

269. A random survey found that 23 out of 200 households were watching channel 10 news last night. If the local market contains 150,000 households, use inference to approximate the total viewership.
**Answer:** $\hat{p} = \frac{23}{200} = .115$, and $(.115)(150,000) = 17250$
equivalently: solve $\frac{23}{200} = \frac{x}{150{,}000}$ to get 17250

270. Suppose the IRS randomly chose 50,000 tax returns to be audited. Out of that sample, 7500 were found to involve serious cheating, averaging \$2700 per case.

   (a) Use inference to estimate the number of cases of cheating among the population of 140 million tax returns.
   **Answer:** $\hat{p} = \frac{7500}{50000} = .15$, and 15% of 140 million is 21 million

   (b) Estimate the total dollar value of the cheating.
   **Answer:** $(21,000,000)(2700) = 56,700,000,000$

271. A scientist tagged 15 bears in the park. Later, she observed that 3 out of 17 bears were wearing tags. Use inference to estimate the number of bears in the park.
**Answer:** solve $\frac{3}{15} = \frac{17}{x}$ to get $x = 85$

272. Suppose a survey of 500 households found that they had an average of $\overline{x} = 1.27$ pets each. If there are 120 million households in the U.S., estimate the total number of pets.
**Answer:** $(1.27)(120) = 152.4$ million

273. A group of 15 girls living in Burnett had 112 pairs of shoes between them.

   (a) For this sample, find $\overline{x}$, the average number of pairs per girl.
   **Answer:** $\overline{x} = \frac{112}{15} = 7.47$ pairs per girl

   (b) If Burnett has a population of 280 girls, use inference to estimate the total number of pairs of shoes.
   **Answer:** $(7.47)(280) = 2091$

274. A running back has run for 220 yards in 3 games. Project his total yardage for the full 16 game season.
**Answer:** solve $\frac{220}{3} = \frac{x}{16}$ to get 1173.

## Confidence Intervals

275. Suppose a 95% confidence interval for $\mu$ is given to be $(78, 92)$. Find the point estimate and the margin of error.
**Answer:** pt est is 85, and MOE is 7

276. Suppose $n = 225$, $\overline{x} = 1642$, and $\sigma = 60$.

   (a) Find the standard error $\sigma_{\overline{x}}$.
   **Answer:** $\frac{60}{\sqrt{225}} = 4$

   (b) Find the 95% confidence margin of error.
   **Answer:** $2\sigma_{\overline{x}} = 8$

   (c) Find a 95% confidence interval for $\mu$.
   **Answer:** $1642 \pm 8 = (1634, 1650)$

277. Write the confidence interval, $(5.61, 5.85)$, in the form "point estimate" $\pm$ "margin of error".
**Answer:** $5.73 \pm 0.12$

278. Write the confidence interval, $215 \pm 25$, in the form $(low, high)$.
**Answer:** $(190, 240)$

279. A survey of college students reported the following hours of sleep per night:

$$\{6.5, 7.2, 8.0, 7.5, 8.2, 6.75, 9.0, 7.8, 7.35\}$$

(a) Enter the data in L1, and find the sample mean.
**Answer:** $\overline{x} = 7.59$

(b) Find a 95% confidence interval for $\mu$.
**Answer:** use tInterval (data) to get $(7.0007, 8.1771)$

280. Suppose you surveyed 150 senior citizens about their sleep habits. On average, they got $\overline{x} = 7.1$ hours of sleep per night, with $s = 0.87$.

(a) Find a 95% confidence interval for $\mu$.
**Answer:** tInterval (stats) gives $(6.96, 7.24)$

(b) State the margin of error.
**Answer:** .14

281. A football player is working out for NFL scouts. He runs the 40-yard dash ten times and records a mean of $\overline{x} = 4.43$ seconds, with $s = .05$.

(a) Find a 90% confidence interval on his 40-yard dash time $\mu$.
**Answer:** tInterval gives $(4.401, 4.459)$

(b) Find a 95% confidence interval on his 40-yard dash time $\mu$.
**Answer:** tInterval gives $(4.394, 4.466)$

(c) Find a 97% confidence interval on his 40-yard dash time $\mu$.
**Answer:** tInterval gives $(4.389, 4.471)$

(d) Find a 99% confidence interval on his 40-yard dash time $\mu$.
**Answer:** tInterval gives $(4.379, 4.481)$

282. Suppose a sample of people that watched Sesame Street as children have an average IQ of 110, with a standard deviation of 18. Compute 95% confidence intervals for $\mu$, and state the margin of error if the sample size was:

(a) $n = 5$
**Answer:** $(87.65, 132.35)$, MOE $= 22.35$

(b) $n = 10$
**Answer:** $(97.12, 122.88)$, MOE $= 12.88$

(c) $n = 50$
**Answer:** $(104.88, 115.12)$, MOE $= 5.12$

(d) $n = 200$
**Answer:** $(107.49, 112.51)$, MOE $= 2.51$

283. Consumer Reports wants to estimate the average battery life of a certain smartphone. In 100 trials, they recorded $\overline{x} = 237$ and $s = 41$ minutes. Write the 98% confidence interval in the form: pointestimate $\pm$ MOE.
**Answer:** tinterval gives $(227.3, 246.7)$, or $237 \pm 9.7$ minutes

284. A simple random sample of 25 Burnett residents have an average of 7.32 shoes, with a standard deviation of $s = 2.87$. If Burnett has a population of 280 girls, use a 95% CI to find lower and upper estimates for the total number of pairs of shoes in Burnett.
**Answer:** tinterval gives $(6.135, 8.505)$, multiply by 280, so so we are 95% sure there are between 1718 and 2381 shoes

285. Here is a stem plot for the birth weight (in ounces) of chimpanzees born in captivity.

| | |
|---|---|
| 3 | 3579 |
| 4 | 01558 |
| 5 | 12333589 |
| 6 | 01345579 |
| 7 | 12258 |
| 8 | 034 |
| 9 | 0 |

Enter the data in L1, and use tInterval to find a 95% confidence interval for $\mu$.

(a) the sample size $n$
**Answer:** $n = 34$

(b) the sample mean $\overline{x}$
**Answer:** $\overline{x} = 59.3$

(c) the sample standard deviation $s$
**Answer:** $s = 15.1$

(d) the 95% CI
**Answer:** $(54.0, 64.6)$

(e) the point estimate
**Answer:** 59.3

(f) the margin of error
**Answer:** 5.28

286. All else being equal, if you increase the sample size, then

(a) does the margin of error increase or decrease?
**Answer:** decrease

(b) does the confidence interval get wider or narrower?
**Answer:** narrower

287. All else being equal, if you increase the confidence level, e.g. from 95% to 99%, then

(a) does the margin of error increase or decrease?
**Answer:** increase

(b) does the confidence interval get wider or narrower?
**Answer:** wider

288. All else being equal, if you increase the standard deviation, then

(a) does the margin of error increase or decrease?
**Answer:** increase

(b) does the confidence interval get wider or narrower?
**Answer:** wider

289. A survey of fifty gamers found that they played a total of 472 hours of video games last week. Let $\mu$ be the average for all gamers. If the observed variance was $s^2 = 38$, find a 95% confidence interval for $\mu$ in the form point estimate $\pm$ MOE.
**Answer:** tinterval with $\overline{x} = \frac{472}{50} = 9.44$ gives $(7.69, 11.19) = 9.44 \pm 1.75$

290. The t-distribution was discovered by William Gosset, an employee of the Guiness brewery in 1908. Suppose the manufacturer wanted a 15 proof alcohol content for its stout beer. Random testing showed that for a sample of $n = 50$ bottles, the alcohol content averaged $\overline{x} = 15.1$ proof with a standard deviation of $s = .38$.

(a) If $\alpha = .05$, find a confidence interval for $\mu$.
**Answer:** tinterval gives $(14.99, 15.21)$

(b) Is the target value of 15 inside the confidence interval?
**Answer:** yes, but just barely

291. Write each confidence interval for a proportion in the form "point estimate" $\pm$ "margin of error".

(a) $(.77, .89)$
**Answer:** $.83 \pm .06$

(b) $(.373, .419)$
**Answer:** $.396 \pm .023$

292. A survey finds that $70\% \pm 4\%$ of teen girls love Justin Bieber.

(a) State the sample/empirical proportion $\hat{p}$.
**Answer:** 0.70

(b) State the margin of error.
**Answer:** 0.04

(c) Write the confidence interval as $(low, high)$.
**Answer:** $(.66, .74)$

293. The Bureau of Labor Statistics is estimating the unemployment rate $p$ in Tennessee. Out of 7200 workforce individuals that were interviewed, 612 of them were unemployed.

(a) State the sample size $n$.
**Answer:** $n = 7200$

(b) Find the sample proportion $\hat{p}$, which is the point estimate for $p$.
**Answer:** $\hat{p} = \frac{612}{7200} = .085$

(c) Use your calculator to find a 95% confidence interval.
**Answer:** 1propZint gives $(.07856, .09144)$

(d) Write the CI as point estimate $\pm$ margin of error.
**Answer:** $.085 \pm .00644$

294. A sociologist wants to estimate the percentage of marriages that end in divorce before the ten year anniversary. In a sample of $n = 5000$ marriages, 31% ended in divorce within ten years.

(a) State the point estimate $\hat{p}$.
**Answer:** $\hat{p} = 0.31$

(b) Out of the sample, how many divorces were recorded?
**Answer:** $(.31)(5000) = 1550$

(c) Find a 95% confidence interval for $p$, and state the margin of error.
**Answer:** 1propZint gives $(.297, .323)$ or $.31 \pm .013$, so the MOE is .013

(d) Find a 99% confidence interval for $p$, and state the margin of error.
**Answer:** 1propZint gives $(.293, .327)$ or $.31 \pm .017$, so the MOE is .017

295. The "confidence level" is one minus the "significance level" ($\alpha$). So if $\alpha = .05$, then the confidence level is 95%.

(a) If $\alpha = .01$, find the confidence level.
**Answer:** 99%

(b) If $\alpha = .03$, find the confidence level.
**Answer:** 97%

(c) If $\alpha = .002$, find the confidence level.
**Answer:** 99.8%

(d) If the confidence level is 98%, find $\alpha$.
**Answer:** $\alpha = .02$

(e) If the confidence level is 99.5%, find $\alpha$.
**Answer:** $\alpha = .005$

(f) If the confidence level is 90%, find $\alpha$.
**Answer:** $\alpha = .10$

296. Suppose that 58% of 500 students that were surveyed prefer Chick-fil-a to Zaxby's.

(a) How many of the sampled students prefer Chick-fil-a ?
**Answer:** $x = (.58)(500) = 290$

(b) Find a 95% CI on the true proportion of students that prefer Chick-fil-a.
**Answer:** Use 1propZint to get $(.537, .623)$.

(c) Find the margin of error.
**Answer:** $.623 - .58 = .043$

(d) Find the MOE for a 99% confidence interval.
**Answer:** the 99% CI is $(.523, .637)$, so the MOE is $.637 - .58 = .057$

(e) Which is wider, the 95% CI or the 99% CI ?
**Answer:** the 99% CI is wider

297. Suppose a sports prognosticator has correctly predicted the winner 56 out of 83 NFL games.

(a) Find $\hat{p}$.
**Answer:** $\hat{p} = 56/83 = .6747$

(b) Find a 95% confidence interval for his long-term success proportion $p$.
**Answer:** 1propZint gives CI $(.57391, .77549)$

298. Suppose that 72 out of 900 participants in a drug trial suffered adverse side-effects. Let $p$ be the population proportion that would have side-effects.

(a) Find a 95% confidence interval for $p$.
**Answer:** 1propZint gives $(.062, .098)$

(b) State the point estimate.
**Answer:** $72/900 = .08$

(c) State the margin of error.
**Answer:** $.018$

299. Suppose that Nielson TV monitors found that 12% of 5000 households watched the World Cup final. There are about 115 million households in the U.S.

(a) Find a 95% confidence interval on the proportion of households that tuned in to the World Cup final.
**Answer:** with $x = (.12)(5000) = 600$, 1propZint gives $(.111, .129)$

(b) That corresponds to an upper estimate of _____ households in the U.S.
**Answer:** $(.129)(115000000) = 14,835,000$

300. A political poll reports that congress has a 20% approval rating, with a MOE of 3%. Assuming $\alpha = .05$,

(a) State $\hat{p}$.
**Answer:** $\hat{p} = .2$

(b) Write the confidence interval.
**Answer:** $(.17, .23)$

(c) Use trial and error to estimate the sample size of this poll.
**Answer:** 1propZint with $x = 140$ and $n = 700$ gives a MOE of .0296, which is pretty close

301. A wildlife researcher wants to estimate the population of bass in a lake. She uses the catch-tag-release-recapture method. Suppose she catches and tags 500 fish, then releases them back into the lake. A week later, she catches 500 more fish and finds that 80 of them have tags.

(a) Find $\hat{p}$, the point estimate for the proportion of fish in the lake that were tagged.
**Answer:** $\hat{p} = \frac{80}{500} = .16$

(b) Find a 95% confidence interval.
**Answer:** 1-propZint gives $(.128, .192)$

(c) Find the corresponding confidence interval for the fish population in the lake.
**Answer:** If the proportion is .128, then $.128x = 80$ gives $x = 625$.
If the proportion is .192, then $.192x = 80$ gives $x = 417$.
So we estimate the population is between 417 and 625 fish.

302. Among a simple random sample of 1000 people, 180 were smokers, and they consumed a total of 2520 cigarettes per day.

(a) What proportion of the sample were smokers?
**Answer:** $\hat{p} = .18$

(b) Find a 95% confidence interval for the population proportion that are smokers.
**Answer:** 1-propZint gives $(.156, .204)$

(c) On average, how many cigarettes per day did the smokers consume?
**Answer:** $\overline{x} = \frac{2520}{180} = 14$

(d) If $s = 20$, find a 95% confidence interval for the average cigarettes per day per smoker.
**Answer:** t-interval with $\overline{x} = 14$, $s = 20$, $n = 180$ gives $(11.1, 16.9)$

## Hypothesis Tests

303. A survey of college students reported the following hours of sleep per night:

$$\{6.5, 7.2, 8.0, 7.5, 8.2, 6.75, 9.0, 7.8, 7.35\}$$

Do a t-test on whether college students get less than the recommended 8 hours of sleep per night.

(a) State the null hypothesis.
**Answer:** $H_0 : \mu = 8$

(b) State the one-tailed alternative hypothesis. Is it left or right tailed?
**Answer:** $H_1 : \mu < 8$, left tailed

(c) State the sample statistic, $\overline{x}$.
**Answer:** $\overline{x} = 7.59$

(d) State the p-value.
**Answer:** t-test gives .073

(e) If $\alpha = .05$, can you reject $H_0$. Is there statistically significant evidence that college students get less than eight hours per night?
**Answer:** $.073 > .05$, so fail to reject $H_0$, not statistically significant

304. McDonald's large French fries are said to contain 500 calories. Run a two-tailed hypothesis test of that claim.

(a) null hypothesis
**Answer:** $H_0 : \mu = 500$

(b) two-tailed alternative hypothesis
**Answer:** $H_1 : \mu \neq 500$

(c) You obtained $n = 20$ samples from area McDonalds, and took them to your lab, where you found $\overline{x} = 512$ and $s = 43$. What is the effect size $\overline{x} - \mu_0$ ?
**Answer:** $512 - 500 = 12$

(d) Find the p-value.
**Answer:** t-test gives .227

(e) Is the difference between $\overline{x} = 512$ and $\mu_0 = 500$ statistically significant?
**Answer:** no, p-value is too big $.227 > .05$, fail to reject $H_0$

305. Suppose that 58% of 500 surveyed students prefer Chick-fil-a to Zaxby's. Do a hypothesis test to investigate whether a majority of all students prefer Chick-fil-a to Zaxby's.

(a) State the null hypothesis. (assume students have no preference)
**Answer:** $H_0 : p = .5$

(b) State the right-tailed alternative hypothesis (a majority prefer Chick-fil-a)
**Answer:** $H_0 : p > .5$

(c) The sample proportion is _____ percentage points above $p_0$.
**Answer:** 8

(d) How many students in the sample prefer Chick-fil-a ?
**Answer:** 58% of 500 is 290

(e) Find the p-value.
**Answer:** 1propZtest gives .000173

(f) If $\alpha = .01$, can you reject $H_0$? Is there a statistically significant preference for Chick-fil-a?
**Answer:** $.000173 < .01$, so reject $H_0$, yes it is statistically significant

306. A rescue squad claims that the average response time for calls received from within the city limits is 5 minutes. However, records show that for the last 40 calls, the sample statistics were $\overline{x} = 6.1$ and $s = 3.5$.

(a) State the null hypothesis.
**Answer:** $H_0 : \mu = 5$

(b) State the one-tailed alternative hypothesis. Is it left or right tailed?
**Answer:** $H_1 : \mu > 5$, right tailed

(c) Find the effect size, $\overline{x} - \mu_0$.
**Answer:** 1.1

(d) State the p-value.
**Answer:** t-test gives .027

(e) State your conclusion if $\alpha = .05$.
**Answer:** $.027 < .05$, so reject $H_0$, the difference is statistically significant

307. A pharmaceutical company is developing an HIV vaccine. Among a certain demographic, suppose the ordinary infection rate is 7 percent. In a field trial, 400 people received the vaccine, and only 17 of them got infected with HIV.

(a) State the sample size.
**Answer:** $n = 400$

(b) If the vaccine were ineffective, how many of the 400 vaccinated people would you expect to be infected.
**Answer:** $(.07)(400) = 28$

(c) State the null hypothesis (that the vaccine is ineffective).
**Answer:** $H_0 : p = 0.07$

(d) State the left-tailed alternative hypothesis (that the vaccine is effective)
**Answer:** $H_0 : p < 0.07$

(e) State the sample proportion $\hat{p}$.
**Answer:** $\hat{p} = \frac{17}{400} = .0425$

(f) Use 1propZtest to find the p-value.
**Answer:** .0156

(g) If $\alpha = 0.05$, would you reject $H_0$ ?
**Answer:** $.015 < .05$, so yes you would reject $H_0$

(h) If $\alpha = 0.01$, would you reject $H_0$ ?
**Answer:** $.015 > .01$, so you would fail to reject $H_0$ at this significance level

(i) Does the vaccine appear to be beneficial? If you needed a lower p-value to get FDA approval for widespread use, what could you do?
**Answer:** The vaccine does appear to reduce the infection rate. The p-value is low (.015), but could be made lower if we increase the sample size and continue to get similar results.

308. At a certain manufacturing plant, the workers are required to have a maximum error rate of 4% on some task. A particular worker has made 21 errors in handling 320 items. As the plant foreman, you may need to reprimand him. First consider the evidence by doing a hypothesis test

(a) If the worker's true error rate was 4%, how many errors would you expect him to make in handling 320 items?
**Answer:** $(.04)(320) = 12.8$

(b) How many did he actually make?
**Answer:** 21

(c) State the null hypothesis. (give the worker the benefit of the doubt)
**Answer:** $H_0 : p = .04$

(d) State the right-tailed alternative hypothesis (that this worker is over the acceptable error rate).
**Answer:** $H_1 : p > .04$

(e) Find the sample proportion.
**Answer:** $\hat{p} = 21/320 = .0656$

(f) Find the p-value.
**Answer:** 1propZtest gives .0097

(g) If $\alpha = .05$, would you reject $H_0$ ?
**Answer:** Yes, $.0194 < .05$, so reject $H_0$

(h) Would you reprimand the worker for having an unacceptably high error rate?
**Answer:** Yes, the p-value is small, indicating that such bad performance is not likely to happen by chance.

309. Suppose you do a right-tailed t-test on $H_0 : \mu = 12$, and the p-value is .00715. State the alternative hypothesis $H_1$. If $\alpha = .01$, is it statistically significant to say that the mean is higher than twelve?
**Answer:** $H_1 : \mu > 12$
yes, since the p-value is less than $\alpha$, it must be that $\overline{x} > 12$ and you would reject $H_0$; the result is statistically significant

310. Would the given p-value lead you to reject $H_0$ or fail to reject $H_0$ at the $\alpha$ significance level?

(a) $H_0 : p = .8$, p-value .73, $\alpha = .05$
**Answer:** $.73 > .05$, fail to reject

(b) $H_0 : \mu = 12$, p-value .03, $\alpha = .05$
**Answer:** $.03 < .05$, reject

(c) $H_0 : p = .90$, p-value .06, $\alpha = .05$
**Answer:** $.06 > .05$, fail to reject

(d) $H_0 : p = .90$, p-value .03, $\alpha = .01$
**Answer:** $.03 > .01$, fail to reject

(e) $H_0 : \mu = 90$, p-value .003, $\alpha = .01$
**Answer:** $.003 < .01$, reject

311. Which would have a lower p-value? See if you can answer without actually doing any calculations.

- $H_0 : p = 0.40$, $\hat{p} = 0.35$, $n = 1000$
- $H_0 : p = 0.40$, $\hat{p} = 0.35$, $n = 1500$
  **Answer:** p-val lower because sample size is bigger

312. Which would have a lower p-value? See if you can answer without actually doing any calculations.

- $H_0 : p = 0.45$, $\hat{p} = 0.35$, $n = 1000$
  **Answer:** p-val lower because there a bigger difference between $\hat{p}$ and $p_0$
- $H_0 : p = 0.40$, $\hat{p} = 0.35$, $n = 1000$

313. Which would have a lower p-value? See if you can answer without actually doing any calculations.

- $H_0 : \mu = 100$, $\overline{x} = 105$, $s = 15$, $n = 50$
- $H_0 : \mu = 100$, $\overline{x} = 93$, $s = 15$, $n = 50$
  **Answer:** p-val lower because $\overline{x}$ is further from $\mu_0$

314. Which would have a lower p-value? See if you can answer without actually doing any calculations.

- $H_0 : \mu = 100$, $\overline{x} = 105$, $s = 15$, $n = 50$
- $H_0 : \mu = 100$, $\overline{x} = 105$, $s = 10$, $n = 50$
  **Answer:** p-val lower because the standard deviation is lower, results are more significant

315. Answer each of the following with "gets smaller", "gets larger", or "doesn't change". Assuming a fixed null hypothesis and point estimate,

(a) if you increase $n$, then the MOE _____.
**Answer:** gets smaller

(b) if you increase $n$, then the $p$-value _____.
**Answer:** get smaller

(c) if you decrease $\alpha$, then the MOE _____.
**Answer:** gets larger

(d) if you decrease $\alpha$, then the $p$-value _____.
**Answer:** doesn't change

316. Guiness wants a certain brew's proof to be $\mu = 15$. William Gosset took fifty samples, and obtained $\overline{x} = 15.1$ with $s = 0.38$.

(a) Do a two-tailed hypothesis test on $H_0 : \mu = 15$. State $H_1$, and find the p-value.
**Answer:** $H_1 : \mu \neq 15$; t-test gives p-value $.0688 > .05$

(b) Now do a one-tailed hypothesis test on $H_0 : \mu = 15$. State $H_1$, and find the p-value.
**Answer:** $H_1 : \mu > 15$; t-test gives p-value $.0344 < .05$

(c) Why do you think the p-value is lower if you do a one-tailed test?
**Answer:** only one tail gets counted, so you get half the probability

317. Suppose a left-tailed proportion test is done on $H_0 : p = .20$, and $\hat{p} = \frac{1}{6}$. The null hypothesis is rejected with $\alpha = .05$. use trial-and-error to find the minimum value of the sample size $n$.
**Answer:** 1propZtest with $x = 65$ and $n = 390$ gives p-value .0499

## Correlation and Matched Pair Tests

318. A researcher is investigating a possible link between apple consumption and doctor visits. If $x$ is the number of apples eaten during a year, and $y$ is the number of doctor visits during that year, the linear regression equation is $y = 2.4 - 0.00512x$, and $r^2 = .0376$. The hypothesis $H_0 : \rho = 0$ has p-value 0.062.

    (a) State the slope.
    **Answer:** $-.00512$

    (b) State the sample correlation.
    **Answer:** $r = -\sqrt{.0376} = -.194$

    (c) Predict the number of doctor visits for someone that eats an apple a day.
    **Answer:** $\hat{y} = 2.4 - .00512(365) = 0.5312$

    (d) Is the correlation between apple consumption and doctor visits statistically significant at the $\alpha = .05$ level?
    **Answer:** no, because $.062 > .05$

319. Here are (made-up) data for student grades according to what row they sit in. (1=front)

| row | grade |
|---|---|
| 1 | 83 |
| 2 | 85 |
| 3 | 82 |
| 4 | 79 |
| 5 | 79 |
| 6 | 76 |

    (a) Use linregTtest to find the correlation between row and grade.
    **Answer:** linregTtest gives $r = -.917$

    (b) State the p-value for $H_0 : \rho = 0$.
    **Answer:** .0102

    (c) Would you reject $H_0$ ?
    **Answer:** yes, since $.0102 < .05$

    (d) Is the correlation between row and grade statistically significant?
    **Answer:** yes

    (e) Can you conclude that sitting in the back row causes a student to get worse grades? Explain.
    **Answer:** The negative correlation is significant, but does not imply causation. It could be that poor students choose to sit in the back, and that they would do the same no matter where they sit.

320. Let $x$ be a C-N student's math ACT score, and let $y$ be his/her math 120 course grade. The linear regression line $y = .3x - 2$ best fits the data collected during recent years. The correlation is $\rho = .63$, and the p-value for $H_0 : \rho = 0$ is $3.4E^{-}4$.

    Is there a "significant" relationship between ACT score and Math 120 grade?
    **Answer:** yes, since the $.00034 < .05$, reject $H_0$; the correlation is significant

321. This table gives matched pair IQ data for twins separated at birth. One twin was adopted, and the other was raised in orphanages.

| adopted | orphanage | difference |
|---|---|---|
| 101 | 107 | -6 |
| 127 | 112 | 15 |
| 110 | 93 | |
| 98 | 98 | |
| 105 | 117 | |
| 118 | 95 | |
| 85 | 91 | |
| 122 | 100 | |
| 103 | 104 | |
| 91 | 82 | |

(a) Fill out the third column for the difference in IQ.

(b) Set up a one-tailed t-test on whether adopted children have higher IQs, report the p-value.
**Answer:** $H_0 : \mu = 0, H_1 : \mu > 0$, p-val .082

322. People tend to exaggerate on self-reported height surveys. Consider this matched pair data of college football players' reported and measured heights in inches.

| reported | measured |
|---|---|
| 73 | 73.25 |
| 75 | 74.50 |
| 70 | 69.25 |
| 76 | 76.00 |
| 69 | 67.75 |
| 75 | 75.25 |
| 72 | 71.25 |
| 77 | 77.50 |
| 75 | 73.50 |
| 72 | 71.50 |
| 70 | 69.75 |

(a) Let $\mu$ be the mean difference between reported and measured heights. Find a 80% CI for $\mu$.
**Answer:** put the differences in L1, then use tInterval to get $(.146, .672)$

(b) Get the p-value for $H_1 : \mu \neq 0$.
**Answer:** t-test gives .058

323. This table shows a weatherman's predicted snowfall versus the actual snowfall for some recent winter storms.

| predicted | actual |
|---|---|
| 5 | 3 |
| 8 | 6 |
| 3 | 7 |
| 5 | 1 |
| 4 | 4 |
| 9 | 5 |
| 2 | 0 |

Does the weatherman tend to have any systematic bias to over or under estimate snowfall? Let $\mu$ be the difference between predicted and actual snowfall. Do a two-tailed matched pair hypothesis test and report the p-value.
**Answer:** First subtract to get the difference $\mu$. The null hyp is $H_0 : \mu = 0$. t-test gives a p-value of .22

## Two Sample Inference

324. A pharmaceutical researcher is testing a new drug that is designed to lower triglyceride levels. Let $\mu_1$ and $\mu_2$ be the decrease in triglyceride levels for high risk individuals that are prescribed the new drug, and the competitor's drug respectively.

| | $n$ | $\overline{x}$ | $s$ |
|---|---|---|---|
| new drug | 75 | 235 | 110 |
| competitor | 45 | 190 | 85 |

(a) Null hypothesis
**Answer:** $H_0 : \mu_1 = \mu_2$

(b) One-tailed alternative hypothesis
**Answer:** $H_1 : \mu_1 > \mu_2$

(c) Effect size $\overline{x}_1 - \overline{x}_2$.
**Answer:** $235 - 190 = 45$

(d) p-value
**Answer:** 2sampTtest gives .00679

(e) If $\alpha = .01$, can you reject $H_0$? Is the difference statistically significant ?
**Answer:** $.00679 < .01$, so reject $H_0$. Yes, the difference is significant.

(f) Get a 95% confidence interval for the difference $\mu_1 - \mu_2$. Write it as point estimate $\pm$ margin of error.
**Answer:** 2sampTint gives $(9.45, 80.55)$, or $45 \pm 35.55$

325. Here are the lifespans (in weeks) of mice that were fed low or high calorie diets. Do a two-tailed hypothesis test on whether the life expectency is the same for both groups.

$$\text{low cal diet: } 98, 85, 91, 68, 87, 102, 77$$

$$\text{high cal diet: } 84, 72, 93, 65, 75$$

(a) null hypothesis
**Answer:** $H_0 : \mu_1 = \mu_2$

(b) alternative hypothesis
**Answer:** $H_1 : \mu_1 \neq \mu_2$

(c) sample means
**Answer:** $\overline{x}_1 = 86.86$ and $\overline{x}_2 = 77.8$

(d) p-value
**Answer:** 2sampTtest gives a p-value of .202

(e) reject $H_0$ ?
**Answer:** $.202 > .05$, so fail to reject $H_0$.

326. In 2008, home teams went 803-513 (wins-losses) in the NBA, and 1368-1090 in MLB. Do a two-tailed hypothesis test of whether the home team winning percentages are identical for the two leagues.

(a) State the null hypothesis.
**Answer:** $H_0 : p_1 = p_2$

(b) State the two-tailed alternative hypothesis.
**Answer:** $H_0 : p_1 \neq p_2$

(c) Find the two sample proportions.
**Answer:** $\hat{p}_1 = \frac{803}{1316} = .610$ and $\hat{p}_2 = \frac{1368}{2458} = .557$

(d) Find the p-value
**Answer:** Use 2propZtest to get a p-value of .0015.

(e) Would you reject $H_0$ at the $\alpha = .01$ significance level?
**Answer:** $.0015 < .01$, so we reject $H_0$. The evidence supports the alternative claim that the home winning percentages are different.

327. A college recently began requiring each student to sign an honor code statement on each test. Before the code was implemented, 3% of 7,000 students were caught cheating. After the code was implemented, only 2% of 3,000 students were caught cheating. Write the null and alternative hypotheses, then find the p-value for a one-tailed test of whether the honor code is reducing incidents of cheating.
**Answer:** if sample 1 is the pre-honor code group, $H_0 : p_1 = p_2$, $H_1 : p_1 > p_2$, 2propZtest p-val .00235

328. Two running backs are competing for the starting job. The average yards per carry are similar, so the coach wants to pick the more consistent player. In scrimmages, Jones has carried the ball 63 times, with a standard deviation of 5.1 yards. Smith has 49 carries with a standard deviation of 4.7 yards. Do a one-tailed hypothesis test to see whether Smith is more consistent.

(a) null hypothesis
**Answer:** $H_0 : \sigma_1 = \sigma_2$

(b) alternative hypothesis
**Answer:** $H_1 : \sigma_1 > \sigma_2$

(c) effect size $s_1 - s_2$
**Answer:** $5.1 - 4.7 = .4$

(d) Report the p-value.
**Answer:** 2sampFtest gives p-value .28

(e) Conclusion. Would you say there is strong evidence that Smith is more consistent?
**Answer:** fail to reject $H_0$; the p-value is not low enough, but the coach should collect some more data to see if the trend holds

329. A new drug is supposed to moderate blood sugar levels. Here are the blood sugar readings for a patient when taking the drug, as opposed to when taking a placebo.

drug: 5, 6, 5, 7, 8, 6, 5, 5

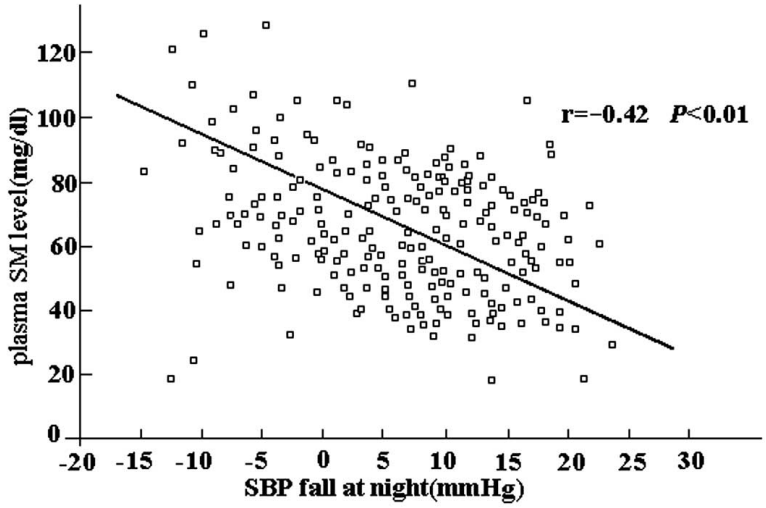
placebo: 3, 7, 4, 5, 11, 8, 6, 4

(a) State the null and alternative hypotheses for a one-tailed F test on whether the patient's blood sugar is moderated while taking the drug.
**Answer:** $H_0 : \sigma_1 = \sigma_2$; $H_1 : \sigma_1 < \sigma_2$

(b) Report the p-value.
**Answer:** 2sampFtest gives .02

# Inference Review

330. Consider this scatterplot of sphingomyelin plasmal levels versus systolic blood pressure fall at night for a set of hypertensive patients.




(a) Estimate the $y$-intercept, i.e. the $y$-value when $x = 0$.
**Answer:** about 80

(b) Estimate the slope.
**Answer:** rise over run with points $(0, 80)$ and $(20, 45)$ gives a slope of $\frac{45-80}{20-0} = -1.75$

(c) Write the equation of the linear regression line.
**Answer:** $y = 80 - 1.75x$

(d) State the correlation.
**Answer:** $r = -.42$

(e) Is the linear relationship statistically significant at the $\alpha = .01$ level ? How do you know?
**Answer:** yes, because we are told that the p-value is $< .01$

331. Starting in the 2011-2012 college basketball season, the NCAA moved the womens' 3-point line back one foot. Did that make a significant difference in the 3-point shooting percentage? Here are data obtained for 150 NCAA division 1 schools.

| season | made | attempted |
|---|---|---|
| 2010-2011 | 29155 | 85379 |
| 2011-2012 | 27636 | 83868 |

(a) State the null hypothesis.
**Answer:** $H_0 : p_1 = p_2$

(b) State the one-tailed alternative hypothesis.
**Answer:** $H_1 : p_1 > p_2$

(c) The effect size is _____ percentage points.
**Answer:** $\hat{p}_1 - \hat{p}_2 = \frac{29155}{85379} - \frac{27636}{83868} = .3415 - .3295 = .012$, so 1.2 percentage points

(d) Find the p-value.
**Answer:** 2propZtest gives p-value .0000000947

(e) Is the difference statistically significant at the $\alpha = .01$ level?
**Answer:** the p-value is less than $\alpha$, so reject $H_0$, the move did make a "significant" difference

(f) Get a 95% confidence interval for the decrease in shooting percentage.
**Answer:** 2propZint gives $(.00746, .01646)$

332. Two competing explorers, Alexandro and Bart, have discovered potential 'fountains of youth' deep in the jungles of southeast Asia. Suppose mice normally have a life span of 80 weeks. Here are the results of the clinical trials on the life span of mice that drank from the respective fountains.

- Alexandro's fountain: $\overline{x} = 82$, $s = 11$, $n = 300$
- Bart's fountain: $\overline{x} = 100$, $s = 25$, $n = 15$

For each explorer, do a one-sample t-test on $H_0 : \mu = 80$.

(a) Statistically speaking, which fountain was found to increase life span more "significantly" ?
**Answer:** p-val for Alex is .0018 ; p-val for Bart is .0079, so statistically speaking, Alexandro's results are more 'significant'

(b) Which fountain appears to have a bigger magnitude of impact on life span ?
**Answer:** Bart's effect size is 20, but Alexandro's is only 2.

333. Waiting times (in minutes) at two doctors' offices were sampled:

$$\text{Doctor Phil: } 17, 22, 31, 21, 24, 28, 19$$

$$\text{Doctor Pepper: } 21, 13, 43, 25, 5, 34, 12, 29$$

Do a one-tailed hypothesis test with $\alpha = .01$ to determine if Doctor Phil's office has more consistent wait times. State the null and alternative hypotheses, find the p-value, and state your conclusion.
**Answer:** $H_0 : \sigma_1 = \sigma_2$ and $H_1 : \sigma_1 < \sigma_2$; 2sampFtest gives p-value $.0184 > .01$, so we fail to reject.

334. Suppose that 40% of Americans will suffer some sort of heart disease. In a study of wine drinkers, only 47 out of 150 suffered heart disease.

(a) Out of 150 Americans, how many would you expect to suffer heart disease?
**Answer:** $(.40)(150) = 60$

(b) Find the point estimate $\hat{p}$ for wine drinkers that suffer heart disease.
**Answer:** $\hat{p} = 47/150 = .313$

(c) Find the 95% confidence interval.
**Answer:** CI $(.2391, .38756)$

(d) Do the hypothesis test $H_0 : p = .4, H_1 : p \neq .4$ using $\alpha = .05$.
**Answer:** p-value $.03026 < .05$, so reject $H_0$

(e) Do the hypothesis test $H_0 : p = .4, H_1 : p \neq .4$ using $\alpha = .02$.
**Answer:** p-value $.03026 > .02$, so fail to reject $H_0$

(f) Do the hypothesis test $H_0 : p = .4, H_1 : p < .4$ using $\alpha = .02$.
**Answer:** p-value $.0151 < .05$, so reject $H_0$

(g) Does the study convince you that drinking wine decreases the risk of heart disease?
**Answer:** No, since rejecting $H_0$ says nothing about causation.

335. A pharmaceutical company has developed a new drug designed to lower cholesterol levels. In the clinical trial, the treatment group received the new drug, while the control group got a placebo. (assume both groups started out having identical cholesterol levels)

| | $n$ | $\overline{x}$ | $s$ |
|---|---|---|---|
| treatment | 130 | 247 | 69 |
| control | 190 | 274 | 55 |

(a) State the null hypothesis.
**Answer:** $H_0 : \mu_1 = \mu_2$

(b) State the one-tailed alternative hypothesis.
**Answer:** $H_0 : \mu_1 < \mu_2$

(c) Find the p-value.
**Answer:** 2sampTtest gives p-value .000122

(d) State your conclusion if $\alpha = .01$.
**Answer:** $.000122 < .05$, so reject $H_0$, the drug's effect is "significant"

(e) Find a 95% confidence interval on the difference.
**Answer:** 2sampTint gives $(-41.28, -12.72)$

336. By definition, the average IQ (interestingness quotient) of the general population is 100. In a sample of 5000 math professors, their average IQ was 99.7, with a standard deviation of 12.

Do a one-tailed hypothesis test to see whether math profs have significantly lower IQ's than the general population.

(a) null hypothesis
**Answer:** $H_0 : \mu = 100$

(b) alternative hypothesis
**Answer:** $H_1 : \mu < 100$

(c) sample statistic
**Answer:** $\overline{x} = 99.7$

(d) p-value
**Answer:** t-test gives .0386

(e) Is the difference "statistically significant" ?
**Answer:** yes, $.0386 < .05$, so reject $H_0$

(f) Is the difference practically significant ?
**Answer:** no, 100 and 99.7 are very close, so not a dramatic difference

337. A school district superintendent wants to quantify the difference in performance between two high schools. Here are the standardized test score summaries:

| | $n$ | $\overline{x}$ | $s$ |
|---|---|---|---|
| North High | 520 | 1080 | 175 |
| South High | 640 | 1010 | 160 |

(a) Find the point estimate for $\mu_1 - \mu_2$.
**Answer:** $1080 - 1010 = 70$

(b) Find the 95% confidence interval for the difference.
**Answer:** 2sampTint gives $(50.5, 89.5)$

(c) What is the margin of error ?
**Answer:** 19.5

(d) Find the p-value for the two-tailed hypothesis test of $H_0 : \mu_1 = \mu_2$.
**Answer:** 2sampTtest gives 3.46E-12, or .00000000000346

(e) Does this prove that North High students are smarter than those at South ?
**Answer:** No, HT does not demonstrate causality. There could be many factors that affect test scores.

338. Here are sampled fitting gaps (in microns) for installed parts on American and Japanese cars:

$$\text{American:} \quad \{123, 170, 85, 226, 150, 93, 179\}$$

$$\text{Japanese:} \quad \{135, 127, 149, 117, 138, 163, 150, 142, 131\}$$

Lower gaps are better for quality assurance.

(a) Find the the sample means $\overline{x}_1$ (American) and $\overline{x}_2$ (Japanese). Which group has smaller fitting gaps?
**Answer:** $\overline{x}_1 = 146.57$ and $\overline{x}_2 = 139.11$, so the Japanese gaps are smaller

(b) Do a one-tailed test of $H_0 : \mu_1 = \mu_2$. If $\alpha = .05$, is the difference "statistically significant" ?
**Answer:** $H_1 : \mu_1 > \mu_2$; 2sampTtest gives a p-value of .357, and since $.357 > .05$, fail to reject $H_0$, not significant

(c) Find the sample standard deviations $s_1$ and $s_2$. Which group has more consistent fitting gaps?
**Answer:** $s_1 = 50.23$ and $s_2 = 13.76$, so the Japanese are more consistent

(d) Do a one-tailed test of $H_0 : \sigma_1 = \sigma_2$. If $\alpha = .05$, is the difference "statistically significant" ?
**Answer:** $H_1 : \sigma_1 > \sigma_2$; 2sampFtest gives p-value $.00088 < .05$, so we reject $H_0$, it is significant

339. In an anonymous survey of 63 C-N students, suppose 24 of them claim to drink alcohol regularly.

(a) State the point estimate for the proportion that drink regularly.
**Answer:** $\hat{p} = 24/63 = .381$

(b) Does a one-tailed hypothesis test with $\alpha = .05$ show that "significantly" more than 30% of C-N students drink regularly?
**Answer:** $H_0 : p = .30$, $H_1 : p > .30$ (rt-tailed), the p-value is $.0804 > .05$, so fail to reject $H_0$, not stat. sig

(c) Does a one-tailed hypothesis test with $\alpha = .05$ show that "significantly" more than 25% of C-N students drink regularly?
**Answer:** $H_0 : p = .25$, $H_1 : p > .25$ (rt-tailed), the p-value is $.0082 < .05$, so reject $H_0$, showing statistical significance; we can confidently say that at least 25% drink regularly

340. A cable company is testing two different letter campaigns designed to entice potential customers to switch from satellite TV. In a certain market, they sent 30,000 letters. Ten thousand received letter "A" while the rest received letter "B". The positive response rates were 1.8% for "A" and 3.1% for "B".

(a) Do a two-tailed hypothesis test to determine if this is a statistically significant difference.
**Answer:** $H_0 : p_1 = p_2$, $H_1 : p_1 \neq p_2$, 2propZtest gives p-value .0000000000447, which is tiny, so there is strong statistical signifcance

(b) Get a 95% confidence interval for the difference in positive response rates.
**Answer:** 2propZint gives $(.0095, .0165)$ or $(.95\%, 1.65\%)$

341. A social media research group wants to know which platform college freshmen consider to be cooler: Facebook or Twitter. Suppose there are 2 million college freshmen in the US, but that only 500 were surveyed - selected randomly from colleges around the country. The survey results showed that 62% of responders thought Twitter was cooler, and the rest thought Facebook was cooler.

(a) State the sample size.
**Answer:** $n = 500$

(b) State the population size.
**Answer:** $2,000,000$

(c) State the sample proportion that preferred Twitter.
**Answer:** $\hat{p} = \frac{310}{500} = .62$

(d) Do a two-tailed test of $H_0 : p = .5$, and state the p-value. Can we say that Twitter is significantly cooler than Facebook?
**Answer:** 1propZtest gives .00000008, so Twitter is statistically significantly cooler

(e) Get a 95% confidence interval for $p$, then write it as point estimate $\pm$ margin of error.
**Answer:** 1propZint gives $(.577, .663)$, or $.62 \pm .043$

(f) Give corresponding low and high values for the number of college freshman that think Twitter is cooler than Facebook.
**Answer:** multiply the CI by 2 million to get low estimate 1,154,000, and high estimate of 1,326,000

342. Here is sample matched pair data for husband and wife body mass index (BMI).

| husband | wife |
|---|---|
| 26 | 18 |
| 23 | 24 |
| 24 | 33 |
| 35 | 32 |
| 20 | 17 |
| 24 | 24 |
| 28 | 36 |
| 40 | 35 |
| 31 | 27 |
| 18 | 22 |
| 33 | 30 |
| 21 | 19 |
| 38 | 26 |

(a) Find $r$, the correlation between husband and wife BMI.
**Answer:** linregttest gives $r = .623$

(b) Find the p-value for $H_0 : \rho = 0$. Is the correlation statistically significant?
**Answer:** .023, yes significant at $\alpha = .05$

(c) Find the average BMI for the husbands $\overline{x}_H$ and the wives $\overline{x}_W$.
**Answer:** $\overline{x}_H = 27.8$ and $\overline{x}_W = 26.4$

(d) Find the p-value for $H_1 : \mu_H \neq \mu_W$. Is the difference statistically significant?
Note, that since these are matched pairs, let $\mu = \mu_H - \mu_W$, and do a t-test on $H_1 : \mu \neq 0$.
**Answer:** make a column for the difference, then ttest gives .414, so not significant

343. Some parents worry that a certain vaccine may increase the risk of autism. Suppose a study tracked 5000 children, 4200 of which had the vaccine. Among the vaccinated group, 50 were diagnosed with autism. Among the un-vaccinated group, only 6 were diagnosed with autism.

(a) State the null hypothesis.
**Answer:** $H_0 : p_1 = p_2$

(b) State the two-tailed alternative hypothesis.
**Answer:** $H_1 : p_1 \neq p_2$

(c) Find the sample proportions.
**Answer:** $\hat{p}_1 = \frac{50}{4200} = .0119$, $\hat{p}_2 = \frac{6}{800} = .0075$

(d) Find the p-value.
**Answer:** 2propZtest gives .278

(e) If $\alpha = .05$, is the autism rate significantly higher in vaccinated children?
**Answer:** not significant, $.278 > .05$, so fail to reject $H_0$, it still would be wise to collect more data

344. A poker player wants to estimate the probability of getting a straight or better hand in Texas Hold-em if the deuces are wild. In simulations, he has achieved such a hand on 273 out of 800 attempts.

(a) State the point estimate (sample proportion)
**Answer:** $\hat{p} = \frac{273}{800} = .34125$

(b) Find a 95% confidence interval on the true probability.
**Answer:** 1propZint gives $(.3084, .3741)$

(c) Find the margin of error.
**Answer:** .03285

345. A nutritionist did a study to see if reduced calorie diets increase lifespan. A control group of 50 mice were given regular diets; they lived an average of 13 months with a standard deviation of 4 months. The experimental group of 30 mice was given a reduced calorie diet; they lived an average of 14.5 months with a standard deviation of 3 months.

Do a two-tailed hypothesis test on $H_0 : \mu_1 = \mu_2$ with $\alpha = .05$.

(a) State the $p$-value.
**Answer:** 2sampTtest gives .061

(b) State your conclusion, and the reason for it.
**Answer:** fail to reject $H_0$ since $.061 > .05$, the difference is not statistically significant

346. In a study of wait times for an oil change, here are the statistics:

| | $n$ | $\overline{x}$ | $s$ |
|---|---|---|---|
| Pep Boys | 19 | 27 | 8 |
| Jiffy Lube | 12 | 24 | 15 |

Do a one-tailed hypothesis test to see if one chain's wait times are signficantly more consistent.

(a) State the null hypothesis.
**Answer:** $H_0 : \sigma_1 = \sigma_2$

(b) State the alternative hypothesis.
**Answer:** $H_1 : \sigma_1 < \sigma_2$

(c) State the p-value.
**Answer:** 2sampFtest .009

(d) Would you reject $H_0$?
**Answer:** yes, since $.009 < .05$

(e) Why are consistent wait times important for somebody who needs an oil change?
**Answer:** So you can plan other errands and not get stuck waiting a long time unexpectedly.

347. Little Caesar's hires a person, call him Al, to stand on the curb of Hwy 11E to advertise the \$ 5 pizza. Over the last year, the store manager collected data on hourly profits (dollars) with and without Al:

| | $n$ | $\overline{x}$ | $s$ |
|---|---|---|---|
| with Al | 387 | 79.41 | 24.3 |
| without Al | 254 | 70.62 | 18.7 |

(a) Find a point estimate for the difference between hourly profits with and without Al.
**Answer:** $79.41 - 70.62 = 8.79$

(b) Find a 95% confidence interval for that difference.
**Answer:** 2samptint gives $(5.44, 12.14)$

(c) If Al makes \$ 6 per hour, do you think he is worth his wages? Are you sure? Explain.
**Answer:** The increase in sales is probably more than Al's wages since $7.79 > 6.00$, but since the 95% CI contains values lower than 6, we can't be that sure.

348. Find $\hat{p}$ for each sample. Which is more "statistically signficant" evidence against $H_0 : p = .5$?

- $x = 14$ out of $n = 20$
**Answer:** $\hat{p} = 14/20 = .7$, p-value .074
- $x = 1050$ out of $n = 2000$
**Answer:** $\hat{p} = 1050/2000 = .525$, p-value .025 (more significant because lower p-value due to much bigger sample)

349. Suppose you have summary statistics $n$, $\overline{x}$, and $s$ for some sample.
Consider doing a two-tailed t-test on $H_0 : \mu = 3$. Suppose $\overline{x} > 3$ and the $p$-value is 0.07.

(a) If $\alpha = .05$, what would be your conclusion?
**Answer:** fail to reject $H_0$

(b) If you increased $n$, with $\overline{x}$ and $s$ unchanged, would it raise or lower the $p$-value?
**Answer:** lower

(c) If you increased $\overline{x}$, with $n$ and $s$ unchanged, would it raise or lower the $p$-value?
**Answer:** lower

(d) If you increased $s$, with $n$ and $\overline{x}$ unchanged, would it raise or lower the $p$-value?
**Answer:** raise

(e) If you switched to a one-tailed test, would it raise or lower the $p$-value ?
**Answer:** lower

350. Suppose in a sample of $n = 12$, you observe $\overline{x} = 25$ with $s = 9$. Find the p-value for

(a) $H_1 : \mu \neq 30$
**Answer:** t-test gives .08

(b) $H_1 : \mu < 30$
**Answer:** t-test gives .04

(c) $H_1 : \mu > 30$
**Answer:** t-test gives .96

351. Alice claims to be able to read Bertha's mind in games of Rock-Paper-Scissors, and wants to prove it by winning more games than would ordinarily be expected. Bertha is skeptical, so her null hypothesis is $H_0 : p = \frac{1}{3}$.

(a) Write Alice's one-tailed alternative hypothesis.
**Answer:** $H_1 : p > .333$

(b) If Alice can win $\hat{p} = \frac{2}{5}$ of the games, use trial-and-error (guess/check) to estimate the sample size required to reach "statistical signficance" at $\alpha = .05$.
**Answer:** 1-propZtest with $x = 54$ and $n = 135$ gives a p-value of .0502

352. Suppose a golfer really is a $\mu = +8$ handicap. For each situation, would you make a Type I error, a Type II error, or neither?

(a) $H_0 : \mu = 8$, $H_1 : \mu < 8$, p-value .183, $\alpha = .05$.
**Answer:** fail to reject $H_0$, so no error made

(b) $H_0 : \mu = 8$, $H_1 : \mu < 8$, p-value .043, $\alpha = .05$.
**Answer:** reject $H_0$, so Type I error

(c) $H_0 : \mu = 8$, $H_1 : \mu < 8$, p-value .043, $\alpha = .01$.
**Answer:** fail to reject $H_0$, no error made

(d) $H_0 : \mu = 10$, $H_1 : \mu < 10$, p-value .183, $\alpha = .05$.
**Answer:** fail to reject $H_0$, so Type II error

(e) $H_0 : \mu = 10$, $H_1 : \mu < 10$, p-value .043, $\alpha = .05$.
**Answer:** reject $H_0$, so no error made

(f) $H_0 : \mu = 10$, $H_1 : \mu < 10$, p-value .043, $\alpha = .01$.
**Answer:** fail to reject $H_0$, so Type II error

353. Suppose a six-sided die is fair, but in a particular run of 200 tosses, you observe 45 ones. If $\alpha = .05$ on a two-tailed hypothesis test, would you commit a Type I error, a Type II error, or neither?
**Answer:** $H_0 : p = \frac{1}{6}$, 1propZtest gives p-value of .027, so you'd reject $H_0$, making a Type I error

354. Suppose an operation's true long-term success rate is 80%. Assuming that $\hat{p} = .80$, consider the two-tailed hypothesis test of $H_0 : p = .75$ with $\alpha = .05$.

(a) If $n = 30$, find the p-value. Would you make a Type II error?
**Answer:** p-val is .527, so you would fail to reject $H_0$, making a Type II error.

(b) If $n = 100$, find the p-value. Would you make a Type II error?
**Answer:** p-val is .248, so you would fail to reject $H_0$, making a Type II error.

(c) Find the minimum sample size required to correctly reject $H_0$.
**Answer:** trial-and-error gives $n = 290$