Test 2 Practice

20. If the Alabama has a 23% chance of winning the championship, then what is the probability that they do not win the championship?
Answer: $1 - .23 = .77$

21. Suppose that you look up the CEO's of 50 large companies, and 36 of them are male. You estimate that the probability that the next CEO you look up is male is $p = \frac{36}{50} = 0.72$. Is this empirical, subjective, or theoretical probability ?
Answer: empirical, based on observational evidence

22. Write the sample space Ω if you flip 3 coins.
Answer: $\Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

23. If you shuffle a deck of cards and draw one at random, what is the probability that it will be either red or a face card?
Answer: equally likely outcomes, so $p = \frac{32}{52} = .615$

24. Suppose I plan a party for statistics students. We will be grilling burgers. Let the random variable X be the number of burgers eaten by a particular student. Previously, I have observed the following empirical distribution:

x	PDF	CDF
0		.08
1		.47
2		.78
3		.96
4		1

(a) Fill out the probability column.

Answer:

x	PDF	CDF
0	.08	.08
1	.39	.47
2	.31	.78
3	.18	.96
4	.04	1

(b) If 50 students will attend the party, find the expected number of burgers that will be eaten in all.
Answer: $\overline{x} = 1.71$ burgers per person; multiply by 50 to get 85.5

25. A tire company offers you a rebate of \$50 if you buy a set of four tires costing \$80 each. It is estimated that 70% of such customers will claim the rebate.

(a) Fill in this probability distribution table, where x is the net revenue for the set of tires.

event	x	$P(x)$
claims rebate		.7
doesn't claim rebate	320	

Answer:

event	x	$P(x)$
claims rebate	270	.7
doesn't claim rebate	320	.3

(b) Find the expected value for the revenue gained by selling a set of four tires.
Answer: $\overline{x} = 285$

26. A basketball player makes 40% of her three-point shots. Each day at practice, she shoots until she has made ten shots. Let the random variable X be the number of shot attempts she will require to make ten shots. Here is an empirical probability distribution generated by computer simulation:

A	B	C	D
X	PDF	CDF	X*PDF
10	0.00012	0.00012	0.00120
11	0.00048	0.00060	0.00528
12	0.00198	0.00258	0.02376
13	0.00594	0.00852	0.07722
14	0.00936	0.01788	0.13104
15	0.01624	0.03412	0.24360
16	0.02416	0.05828	0.38656
17	0.03346	0.09174	0.56882
18	0.04276	0.13450	0.76968
19	0.05202	0.18652	0.98838
20	0.05856	0.24508	1.17120
21	0.06274	0.30782	1.31754
22	0.06772	0.37554	1.48984
23	0.06802	0.44356	1.56446
24	0.06628	0.50984	1.59072
25	0.06588	0.57572	1.64700
26	0.06166	0.63738	1.60316
27	0.05554	0.69292	1.49958
28	0.05020	0.74312	1.40560
29	0.04322	0.78634	1.25338
30	0.03734	0.82368	1.12020
31	0.03364	0.85732	1.04284
32	0.02848	0.88580	0.91136
33	0.02328	0.90908	0.76824
34	0.01874	0.92782	0.63716
35	0.01600	0.94382	0.56000
36	0.01170	0.95552	0.42120
37	0.00988	0.96540	0.36556
38	0.00770	0.97310	0.29260
39	0.00622	0.97932	0.24258
40	0.00486	0.98418	0.19440
41	0.00366	0.98784	0.15006
42	0.00320	0.99104	0.13440
43	0.00232	0.99336	0.09976
44	0.00138	0.99474	0.06072
45	0.00142	0.99616	0.06390
46	0.00100	0.99716	0.04600
47	0.00070	0.99786	0.03290
48	0.00058	0.99844	0.02784
49	0.00060	0.99904	0.02940
50	0.00034	0.99938	0.01700
51	0.00020	0.99958	0.01020
52	0.00016	0.99974	0.00832
53	0.00006	0.99980	0.00318
54	0.00002	0.99982	0.00108
55	0.00006	0.99988	0.00330
56	0.00004	0.99992	0.00224
57	0.00002	0.99994	0.00114
58	0.00002	0.99996	0.00116
59	0.00000	0.99996	0.00000
60	0.00000	0.99996	0.00000
61	0.00002	0.99998	0.00122
62	0.00000	0.99998	0.00000
63	0.00002	1.00000	0.00126
total	1.00000		24.98924

(a) Find the expected value of X.
Answer: $\overline{x} = 24.98924$

(b) Find the 5 number summary.
Answer: $(10, 21, 24, 29, 63)$

(c) The 37th percentile is _____.
Answer: $x = 22$

(d) $x = 37$ is the _____ percentile.
Answer: 96th

(e) Is it conceivable that $x = 65$ could occur in the future?
Answer: Yes. Even though it didn't happen empirical simulation, it could still happen in the future.

(f) Find $P(X = 20)$.
Answer: $PDF(20) = .05856$

(g) Find $P(X < 20)$.

Answer: $CDF(19) = .18652$

(h) Find $P(X > 20)$.
Answer: $1 - CDF(20) = 1 - .24508 = .75492$

(i) Find $P(X \leq 20)$.
Answer: $CDF(20) = .24508$

(j) Find $P(X \geq 20)$.
Answer: $1 - CDF(19) = 1 - .18652 = .81348$

(k) Find $P(20 \leq X \leq 30)$.
Answer: $CDF(30) - CDF(19) = .82368 - .18652 = .63716$

27. Suppose that you play a game in which you roll a die and flip a coin. You get to move the number of spaces shown on the die, but doubled if the coin lands heads. So if you rolled a 4 and flipped heads (4H), you'd move 8 spots. Let the random variable x be the number of spots you get to move.

(a) Write the sample space Ω.
Answer: $\Omega = \{1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T\}$

(b) Create the probability distribution for x.

Answer:

x	P(x)
1	1/12
2	2/12
3	1/12
4	2/12
5	1/12
6	2/12
8	1/12
10	1/12
12	1/12
	1

(c) What is the theoretical probability that you get to move at least 6 spots ?
Answer: 5/12

(d) Find the expected value of x.
Answer: $\mu = \frac{63}{12} = 5.25$

28. Suppose $P(A) = .75$ and $P(B) = .44$, and the events are independent. Find the probability of neither A nor B.
Answer: $(1 - .75)(1 - .44) = (.25)(.56) = .14$

29. At a Christmas party, the host hands out gift bags. There are 15 gift bags; four of them contain a chocolate snowman, and the others contain a lump of coal.

If you and your date each select a bag at random, find the probability that you receive different gifts.
Answer: you could get snowman and your date coal, or vice-versa, so $(11/15)(4/14)+(4/15)(11/14) = .42$

30. Suppose a free throw shooter goes to the line for two shots. He has a 60% chance of making both. Find the probability that he makes exactly 1 of 2.
Answer: First find the probability for making one shot, $p^2 = .6$, so $p = .7746$. Now he could either make-miss or miss-make. So the answer is $p(1 - p) + (1 - p)p = .349$.

31. Consider being dealt four cards from a standard 52 card deck.

(a) Find the probability that they are all spades.
Answer: $(13/52)(12/51)(11/50)(10/49) = .00264$

(b) Find the probability of getting at least one spade.
Answer: $1 - (39/52)(38/51)(37/50)(36/49) = .696$

(c) Find the probability of getting exactly one spade.
Answer: there are four ways this could occur: SNNN,NSNN,NNSN,NNNS, so the answer is $4(13/52)(39/51)(38/50)(37.49) = .439$

32. Suppose Alan, Brad, and Carl are racing each other in a 100 meter dash. Alan has a 70% chance of beating Brad. Alan has a 40% chance of beating Carl.

    Explain why the probability that Alan wins the race by beating both Brad and Carl is not $(.70)(.40) = .28$. Do you think this probability should be higher or lower than .28 ?
    Answer: Beating Brad and beating Carl are not independent events. Given that Alan beats Brad, there must be an adjustment to the probability that he also beats Carl. If Alan beats Brad, then we would assume that Alan ran fast and therefore his probability of beating Carl would increase. So the probability that Alan wins the race is higher than .28, but it's hard to estimate how much higher without more information.

33. Suppose that if you park illegally at Carson-Newman, you have a 8% chance of getting a ticket. If you park illegally 15 times during a semester, then

    (a) What distribution models X, the number of tickets you will get?
    Answer: $X \sim BI(15, .08)$

    (b) what is the expected number of tickets you will get?
    Answer: $\mu = np = 15(.08) = 1.2$ tickets

    (c) what is the probability that you don't get any tickets?
    Answer: $\mathit{bipdf}(15, .08, 0) = .286$

    (d) what is the probability that you get more than one ticket?
    Answer: $1 - \mathit{bicdf}(15, .08, 1) = .340$

34. A 75% free throw shooter takes 100 shots. Let X be the number of shots she makes.

    (a) Find the mean.
    Answer: $\mu = (0.75)(100) = 75$

    (b) Find the standard deviation.
    Answer: $\sigma = \sqrt{(100)(.75)(.25)} = 4.33$

    (c) Find the probability that she makes exactly 75 shots.
    Answer: $\mathit{bipdf}(100, .75, 75) = .0918$

    (d) Find the probability that she makes at least 80 shots.
    Answer: $1 - \mathit{bicdf}(100, .75, 79) = .149$

    (e) Find $P(70 \leq X \leq 80)$.
    Answer: $\mathit{bicdf}(100, .75, 80) - \mathit{bicdf}(100, .75, 69) = .797$

35. Explain why the binomial distribution is a bad model for the number of C-N students to catch the flu this winter.
    Answer: flu is contagious, so we're dealing with non-independent events

36. If $X \sim BI(20, .4)$ and $Y \sim BI(30, .3)$, find the probability that $X = 8$ and $Y \leq 10$.
    Answer: multiply the probabilities to get $\mathit{bipdf}(20, .4, 8) \cdot \mathit{bicdf}(30, .3, 10) = .13125$

37. If a population is distributed $N(50, 6)$, then about 95% of individuals will fall between ____ and ____.
    Answer: 2 standard deviations to each side, between 38 and 62

38. Suppose the height of Amazon women is $X \sim N(70, \sigma)$ inches.

    (a) Gryne's is 6'3" tall, and has a z-score of 1.43. Find σ.
    Answer: solve $\frac{75-70}{\sigma} = 1.43$ to get $\sigma = 3.5$ inches

(b) Find the 65th percentile height.
Answer: $invnorm(.65, 70, 3.5) = 71.35$ inches

(c) At what percentile is 65 inches?
Answer: $normcdf(-E99, 65, 70, 3.5) = .076$, so approx the 8th percentile

(d) In a tribe of 200 Amazons, how many do you expect to be over 6 feet tall?
Answer: $normcdf(72, E99, 70, 3.5) = .284$, so $(.284)(200) \approx 57$

39. Suppose the length of a statistics lecture is a random variable having the distribution $N(70, 7)$ minutes, and the scheduled class period is 75 minutes.

(a) What is the probability that a class lets out late?
Answer: $normcdf(75, E99, 70, 7) = .2375$

(b) Find the inter-quartile range of the lecture length.
Answer: $Q_3 - Q_1 = invnorm(.75, 70, 7) - invnorm(.25, 70, 7) = 9.44$

(c) What is the probability that class lets out late fewer than 5 times during a 30 lecture semester?
Answer: $bicdf(30, .2375, 4) = .127$

(d) How many minutes did a lecture last if its z-score is 1.3 ?
Answer: $\frac{x-70}{7} = 1.3$, so $x = 79.1$ minutes

40. If $\sigma = 20$, find the standard error in measuring $\overline{x}$, the average of $n = 100$ observations.
Answer: $\sigma_{\overline{x}} = \frac{20}{\sqrt{100}} = 2$

41. Tiffany has a 1000 minutes per month cellphone plan. Let the random variable X be the length of a single conversation. Suppose $\mu = 7$ and $\sigma = 13$.

(a) Do you think the distribution of X is normal? Explain.
Answer: No, a conversation can't have negative length, and $\sigma > \mu$. Furthermore, there will be more short/moderate phone calls, and a few really long ones, so the distribution will be right skewed.

(b) Suppose Tiffany makes 121 phone calls in a month. According to the central limit theorem, the average conversation length, $\overline{x}$, has approximately what distribution?
Answer: $N(7, 13/\sqrt{121}) = N(7, 1.18)$

(c) Find the probability that she goes over her 1000 minute allotment with her 121 conversations.
Answer: This equates to $1000/121 = 8.264$ min/convo, so $normcdf(8.264, E99, 7, 1.18) = .142$

42. Jeffrey Lebowski's bowling game score is modeled by the distribution $x \sim N(185, 23)$.

(a) Find the probability he scores at least 200 pins. (use a continuity correction)
Answer: $normcdf(199.5, E99, 185, 23) = .264$

(b) Find his 95th percentile score.
Answer: $invnorm(.95, 185, 23) = 222.83$

(c) During a tournament, he bowls 12 games. Find the probability that he bowls at least 200 at least three times.
Answer: $1 - bicdf(12, .264, 2) = .651$

(d) During a tournament, Lebowski bowls 12 games. According to the Central Limit Theorem, what is the distribution of his average score, $\overline{x}$.
Answer: $N(185, 23/\sqrt{12}) = N(185, 6.64)$

(e) Find the 75th percentile for Lebowski's average score in a 12 game tournament.
Answer: $invnorm(.75, 185, 23/\sqrt{12}) = 189.5$

(f) Find the probability that he averages at least 200 pins per game in a 12 game tournament.
Answer: Using the CLT, $normcdf(200, E99, 185, 23/\sqrt{12}) = .012$

(g) The bowling league commissioner assigns handicaps based on each player's sample mean score, $\overline{x}$. How many games must Lebowski roll to make the standard error $\sigma_{\overline{x}} \leq 2$?
Answer: solve $23/\sqrt{n} \leq 2$ to get $n \geq 132.25$ so $n = 133$

43. Suppose an indoor cat's lifespan is the random variable $x \sim N(13, 3)$ years. A litter of 5 kittens was born, and you decided to keep them all. Assume the cats' lifespans are independent.

(a) What is the life "expect"ancy of a cat?
Answer: the "expect"ancy is the mean, or 13 years.

(b) Find the probability that an individual cat will live to be at least 15 years old.
Answer: $normcdf(15, E99, 13, 3) = .2525$

(c) Find the 99th percentile cat lifespan.
Answer: $invnorm(.99, 13, 3) = 19.98$ years

(d) Write the distribution of $\overline{x}$, the average lifespan of your 5 cats.
Answer: $\overline{x} \sim N(13, 3/\sqrt{5})$

(e) Find the probability that the cats will live an average of 15 years or more.
Answer: $normcdf(15, E99, 13, 3/\sqrt{5}) = .068$

(f) Find the probability that a majority of the cats survive to be 15 years old.
Answer: majority means at least 3 out of 5, so $1 - bicdf(5, .2525, 2) = .106$

44. A group of people went on a diet for three months. This data records each participant's average daily exercise, and how much weight he/she lost.

exercise (x)	weight loss (y)
10	8
30	17
15	12
40	19
12	10
45	15
25	14
60	23
15	5

(a) Find the correlation between x and y.
Answer: linregttest gives $r = .884$

(b) Find the linear regression line for this data.
Answer: $y = 5.62 + .29x$

(c) Predict the weight loss for a person that exercised 20 minutes a day on this diet.
Answer: $\hat{y} = 5.62 + .29(20) = 11.4$ pounds

45. Let x be a student's ACT score, and y be his/her corresponding SAT score. Suppose the linear regression line is

$$y = 950 + 40(x - 20)$$

(a) What is the slope of the regression line?
Answer: 40

(b) If a student scored a 27 on the ACT, what is the corresponding SAT score ?
Answer: $\hat{y} = 150 + 40(27) = 1230$

46. If $y = 7.15 - 0.58x$ is the linear regression line for a scatterplot with $r^2 = .1225$, then what is the correlation between x and y?
Answer: the slope is negative, so $r = -\sqrt{.1225} = -.35$