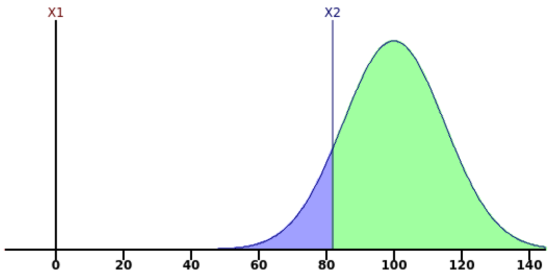
Thu, Mar 19 - Slide 205

1. We know that $\mu = 100$ and $\sigma = 15$.



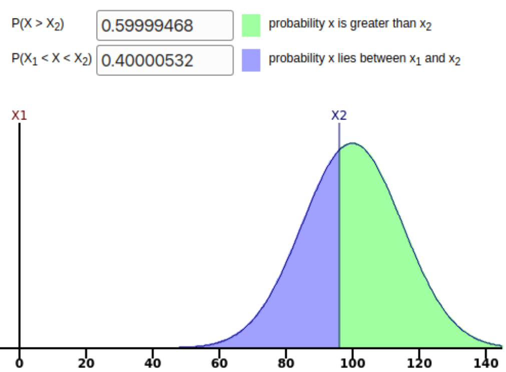
Plug into the z-score formula:

$$-1.2 = \frac{x - 100}{15}$$

solve to get an IQ score of $x = 82$. This corresponds to the 11.5 percentile.

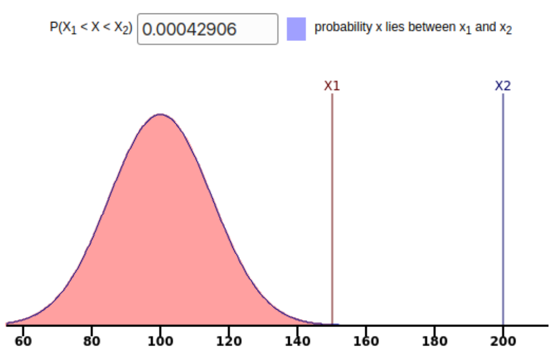
$$normcdf(0, 82, 100, 15) = .115$$

2. Sketch a picture. This is asking for the 40th percentile.



$$invnorm(.40, 100, 15) = 96.2$$

3. Sketch a picture. 150 and higher is the far right tail.



$$normcdf(150, 9999, 100, 15) = 4.29E - 4 = .000429$$

So a tiny proportion of the population has IQ this high. To estimate how many individuals we are talking about, multiply by the population size (U.S. about 325 million).

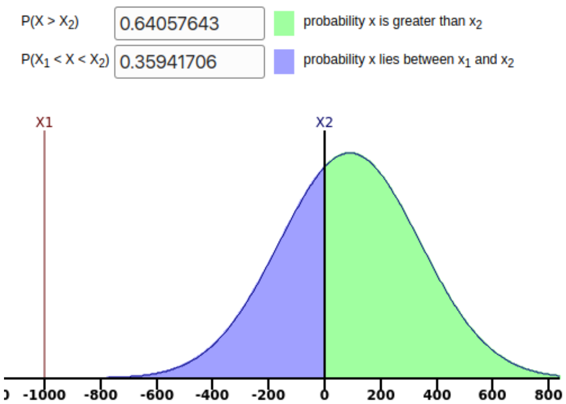
$$(.0000429)(325,000,000) \approx 139000$$

4. nevermind

Thu, Mar 19 - Slide 206

Zero is the dividing line. Say $F < 0$ are fiends, and $F > 0$ are friends.

1. Sketch a picture.

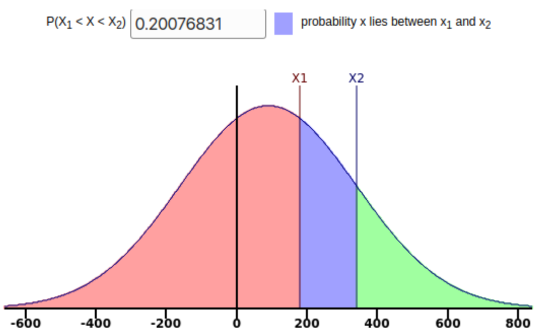


$$normcdf(0, 9999, 90, 250) = .641$$

so 64.1% are friendly and 35.9% are fiends

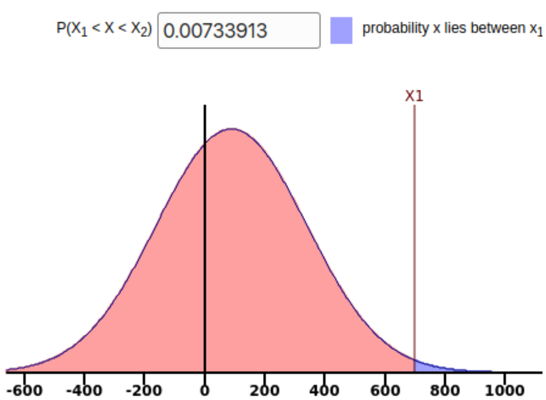
2. nevermind

3. Just use the calculator



$$normcdf(180, 340, 90, 250) = .201$$

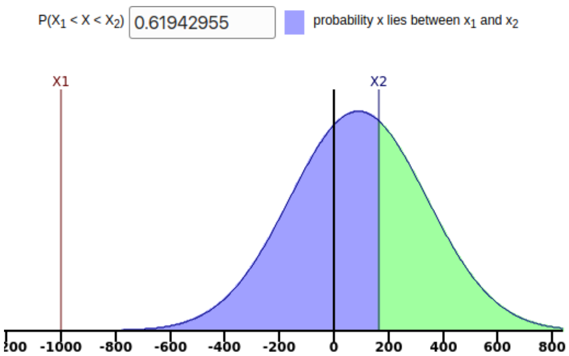
4. The probability that a ghost is more friendly than Casper is



$$normcdf(700, 9999, 90, 250) = .00734$$

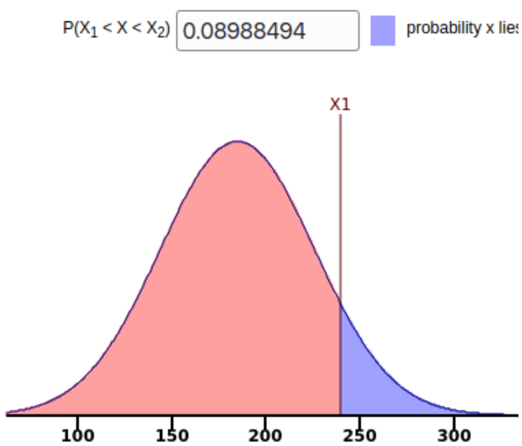
Since $1/.00734 = 136$, we could say 1 out of 136 ghosts is friendlier than Casper.

5. $x = invnorm(.62, 90, 250) = 166$, and $z = \frac{166-90}{250} = .304$



Thu, Mar 19 - Slide 207

1. Sketch a picture.



The information given says that the 91st percentile is 240, so we want

$$invnorm(.91, 185, \sigma) = 240$$

Use your calculator to guess, e.g. if you guess $\sigma = 25$, you'll get an answer too low. So adjust your guess until you find that $\sigma = 41$ works.

2. Multiply the proportion times the population size.

$$normcdf(200, 240, 185, 41) * 5000 = 1340$$

So about 1340 of those people have moderately high cholesterol.

Thu, Mar 19 - Slide 209

8 A single cat, weighing x pounds, cannot be under 9 pounds and also at least 10 pounds, so the probability is zero.

9 Two different (independent) cats: one under 9 lbs, and the other at least 10 lbs. So just multiply:

$$normcdf(-999, 9, 8, 2) * normcdf(10, 999, 8, 2) = (.691)(.159) = .11$$

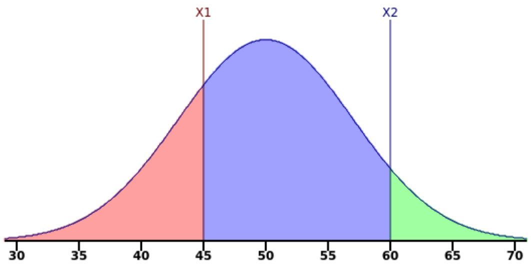
Thu, Mar 19 - Slide 210

3 Pick any μ and σ you want, e.g. $\mu = 50$ and $\sigma = 10$.

$$x = invnorm(.80, 50, 10) = 58.42$$

$$z = \frac{58.42 - 50}{10} = .842$$

4 Sketch a picture.



Either add the disjoint pink plus green regions, or complement the purple region.

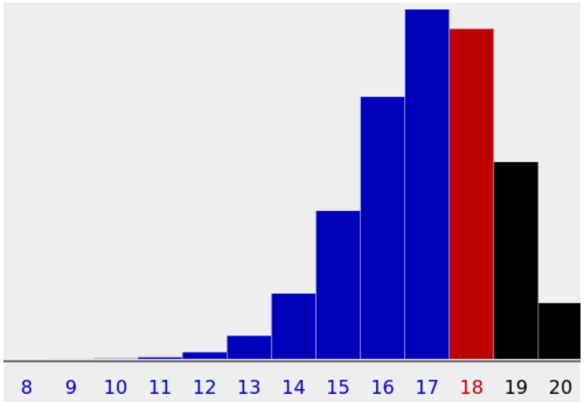
$$\begin{aligned} P(x < 45) + P(x \geq 60) &= normcdf(-999, 45, 50, 7) + normcdf(60, 999, 50, 7) \\ &= 1 - normcdf(45, 60, 50, 7) \\ &= .314 \end{aligned}$$

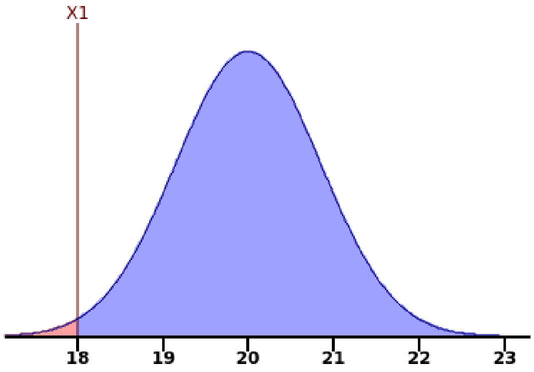
Thu, Mar 19 - Slide 211

We have learned two important theoretical distributions:

- binomial $BI(n, p)$ where n is how many attempts, and p is the probability of success
- normal $N(\mu, \sigma)$, where μ is the mean and σ is the standard deviation

Let's contrast them with the given example.





	X	Y
discrete or continuous	discrete	continuous
mean	$(20)(.85) = 17$	20
standard deviation	$\sqrt{20(.85)(.15)} = 1.60$	.85
prob. equals 18	$bipdf(20, .85, 18) = .2293$	zero
prob. more than 18	$1 - bicdf(20, .85, 18) = .1756$	$normcdf(18, 999, 20, .85) = .991$
prob. at least 18	$1 - bicdf(20, .85, 17) = .4041$	$normcdf(18, 999, 20, .85) = .991$

Thu, Mar 19 - Slide 214

- Let individuals x from a distribution have mean μ and standard deviation σ.
- Select a random sample of n individuals, so that $\overline{x}$ is the sample mean.
- Then each sample will have a different $\overline{x}$; it's the luck of the draw.
- But, the tendency is for the high and low values in your sample to "average out".
- As sample size n increases, the law of large numbers describes how $\overline{x}$ will generally get closer to μ.
- We can prove a mathematical formula for how the standard deviation of $\overline{x}$ drops as n increases.

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

  This is called the **standard error** of $\overline{x}$.

Thu, Mar 19 - Slide 216

Let $\sigma = 5.6$ be the standard deviation for $n = 1$ round of golf.

n	$\sigma_{\overline{x}}$
1	$5.6/\sqrt{1} = 5.60$
2	$5.6/\sqrt{2} = 3.96$
3	$5.6/\sqrt{3} = 3.23$
4	$5.6/\sqrt{4} = 2.80$
5	$5.6/\sqrt{5} = 2.50$
6	$5.6/\sqrt{6} = 2.29$
7	$5.6/\sqrt{7} = 2.12$
8	$5.6/\sqrt{8} = 1.98$
9	$5.6/\sqrt{9} = 1.87$

So with $n = 8$ rounds of golf, the standard error drops below 2 strokes.