Ratios / Proportions / Percents

- In a standard 52-card deck, what proportion of cards have faces on them? Write your answer as a reduced fraction, a decimal, and as a percent.
 Answer: p = ¹²/₅₂ = ³/₁₃ ≈ 0.231 = 23.1%
- 2. A bag of 250 M&M's has 57 blue pieces.
 - (a) What proportion are blue? **Answer:** $\frac{57}{250} = .228$
 - (b) What proportion are not blue? **Answer:** $\frac{193}{250} = 1 - .228 = .772$
- 3. Suppose the population of a city went from 50,000 to 57,000.
 - (a) The absolute change is _____ people. Answer: 7000
 - (b) The relative change is _____ percent. Answer: 7000/50000 = .14 = 14%
- 4. Suppose the number of murders in a city decreased from 34 last year to 22 this year. That is a _____% drop.

Answer: 12/34 = .353 = 35.3%

- 5. Suppose a company's revenues were \$100 million three years ago. During the last three years, revenues increased by 20%, decreased by 15%, and then increased by 31%.
 - (a) What was the revenue this year?
 Answer: 100(1.2)(.85)(1.31) = 133.6 million
 - (b) Would it have been better to increase revenues by a consistent 12% each of the three years? **Answer:** $100(1.12)^3 = 140.5$, so yes
- 6. Suppose 319 out of 435 congressmen voted for a particular bill. The rest voted against it.
 - (a) The proportion voting for the bill was.Answer: 319/435 = .733
 - (b) The percentage voting against the bill was. **Answer:** 1 - .733 = .267 = 26.7%
 - (c) The for:against ratio was ______ to 1. Answer: 319:116 = 11:4 = 2.75:1
 - (d) The number of votes for the bill was _____% times the number of votes against it.
 Answer: 275%
 - (e) ____% more congressmen voted for the bill than voted against it. Answer: $\frac{203}{116} = 175\%$
 - (f) ____% fewer congressmen voted against the bill than voted for it. Answer: $\frac{203}{319} = 63.6\%$
- 7. Suppose the Dollar:Euro exchange rate is 138:100. That means that 138 dollars can be exchanged for 100 Euros. How many Euros would you get for \$ 500? **Answer:** solve $\frac{138}{100} = \frac{500}{x}$ to get 362.2 Euros
- 8. At a certain university, the athlete graduation rate has gone from 50% to 55%. You would say the graduation rate has gone up _____ percentage points, which represents a _____ percent increase. Answer: 5 percentage points, 10% increase

9. Luke 21:1 And Jesus looked up and saw the rich putting their gifts into the treasury, 2 and He saw also a certain poor widow putting in two mites. 3 So He said, Truly I say to you that this poor widow has put in more than all; 4 for all these out of their abundance have put in offerings for God, but she out of her poverty put in all that she had.

Compare these gifts as a percentage of the giver's wealth.

(a) Suppose a rich man has \$ 1,000,000 and gives a donation of \$ 50,000. What percentage of his wealth did he give?
 A neuron 50000 = 05 = 5%

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Answer: \frac{50000}{1000000} = .05 = 5\%
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- (b) A poor woman has \$ 2 and gives it all. What percentage of her wealth did she give? **Answer:** $\frac{2}{2} = 1 = 100\%$
- 10. An animal shelter is currently holding 21 dogs. If the proportion of shelter animals that are dogs is p = .583, then how many animals are in the shelter? **Answer:** solve $.583 = \frac{21}{n}$ to get n = 36
- 11. Your current salary is \$ 30,000. A certain promotion comes with a 20% raise.
 - (a) How much money is the raise? **Answer:** Then "20% of 30,000" equals (20%)(30000) = (.20)(30000) = 6000 dollars
 - (b) What would be your salary after the promotion? **Answer:** 30000 + (20%)(30000) = (120%)(30000) = (1.20)(30000) = 36000
- 12. A college had 515 students in last year's freshman class. Of those, 335 returned for their sophomore year, while the rest dropped out or transfered.



- (a) Find the "retention rate", which is the proportion of returning students. **Answer:** $p = \frac{335}{515} = .65$
- (b) This year's freshman class had 508 students. How many students must return to hit the target retention rate of 70% ?
 Answer: (0.70)(508) = 356
- An athlete told the reporter that she gave 110% effort. Is that possible?
 Answer: Percentage effort given is a proportion of her capability. It is impossible to give more than everything she had.
- 14. A sweater was marked down 40% so that the price you paid was \$25. What was the original price? **Answer:** Let x be the original price. You paid 100% - 40% = 60% of x, so solve .60x = 25 to get x = 41.67
- 15. At a certain college, the ratio of girls to guys is 1.38 to 1.



- (a) What proportion of the students are girls?
 Answer: An equivalent ratio is 138 : 100, so 138 girls out of 238 students. So the proportion of girls is ¹³⁸/₂₃₈ = .58
- (b) What proportion of the students are guys? **Answer:** $\frac{100}{238} = .42$
- (c) If there are 1200 girls, how many guys? **Answer:** solve $\frac{1200}{x} = \frac{138}{100}$ to get x = 870
- (d) There are ____% more girls than guys. Answer: guys are the reference point, so $\frac{138-100}{100} = .38$, so 38% more girls
- (e) There are ____% fewer guys than girls. **Answer:** girls are the reference point, so $\frac{100-138}{138} = .275$, so 27.5% fewer guys
- 16. In a bag of candy, the ratio of Snickers to Reese's is 1.8:1. If there are a total of 182 pieces, fill in the blanks:
 - (a) There are _____ Snickers and _____ Reece's. Answer: Write the ratio Snickers:Reeces as 1.8x : x where 2.8x = 182. Then x = 65 so 117 Snickers and 65 Reeces
 - (b) $\[\% \] %$ of the candy is Snickers, and $\[\% \] %$ is Reece's. Answer: 64.3, 35.7
 - (c) There are _____ % more Snickers than Reece's. Answer: 80
 - (d) There are _____ % fewer Reece's than Snickers. Answer: 44.4
- 17. Suppose congress raises your income tax rate from 27% to 33%. This is a _____ percentage point increase, and your tax rate has increased by _____ percent.
 Answer: 33 27 = 6 percentage point increase. 33-27/27 = .222, so your tax rate increased by 22.2 percent
- 18. Here is a basketball team's shooting by half:

	\mathbf{FG}	percentage
1 st	25/40	62.5%
2nd	10/30	33.3%
game	35/70	

- (a) What was the team's shooting percentage for the game? Answer: 35/70 = .50 = 50%
- (b) Does that equal the average of 62.5% and 33.3% ? **Answer:** no, $\frac{62.5+33.3}{2} = 47.9$

- 19. You got a job making \$100,000.
 - (a) After one year, they gave you a 40% raise. Now what is your salary?Answer: (1.4)(100,000) = 140,000
 - (b) After the second year, they have to cut your salary by 20%. Now what is your salary?
 Answer: (.8)(140,000) = 112,000
 - (c) Is it true that after a 40% raise and then a 20% cut, you are still up by 20% ?
 Answer: no, you are up by only 12%

Basic Statistics

- 20. Would data for the random variable be considered quantitative or qualitative? discrete or continuous?
 - (a) speed of a car Answer: quant, cont
 - (b) number of free throws made **Answer:** quant, disc
 - (c) tree circumference Answer: quant, cont
 - (d) brand of jeans Answer: qual, disc

- (e) gallons of gas Answer: quant, cont
- (f) nut/bolt size Answer: quant, disc
- (g) social security number Answer: qual, disc
- (h) lifespan of dog Answer: quant, cont
- 21. At Walmart, there is a checkout line with this sign posted:



- (a) Is the number of items in your cart discrete or continuous? Answer: discrete
- (b) What English word should be used instead of "less" ? Answer: fewer
- 22. Suppose you are taking 6 classes this semester, and your class sizes are:

$$\{7, 10, 11, 16, 19, 52\}$$

- (a) Find your mean class size. Include units. **Answer:** $\overline{x} = 115/6 = 19.17$ students per class
- (b) Find your median class size. **Answer:** $\frac{11+16}{2} = 13.5$ students per class
- (c) Find the range of this data set. Answer: 52 - 7 = 45

23. Here are Tony Stwart's race finishes during his historic run in the 2011 NASCAR Championship Chase.

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\{1, 1, 25, 15, 8, 7, 1, 1, 3, 1\}
```

- (a) Find the sample size. **Answer:** n = 10
- (b) Find the total $\sum x$. Answer: 63
- (c) Find the mean \overline{x} . **Answer:** $\frac{63}{10} = 6.3$
- (d) Find the median. Answer: (1+3)/2 = 2
- (e) Find the mode. Answer: 1
- (f) Find the range. Answer: 25 - 1 = 24
- 24. Find the basic statistics for this sample of movie run-times: {1:48, 2:05, 1:34, 1:52, 2:21}. (first convert to minutes)
 - (a) sample size Answer: n = 5
 - (b) total **Answer:** $\sum x = 580$
 - (c) mean **Answer:** $\overline{x} = 116$ minutes, or 1:56 (d) median
 - **Answer:** 112 minutes (1:52)
 - (e) range Answer: 47
- 25. Suppose you have a list of 24 numbers with $\sum x = 420$. Find the mean \overline{x} . Answer: $\overline{x} = \frac{420}{24} = 17.5$
- 26. Suppose 18 children ate a total of 77 pieces of candy. Then the mean is _____ pieces per child. Answer: 77/18 = 4.28
- 27. Suppose n = 27 students averaged \$ 9.63 of cash. How much total cash, $\sum x_i$, do they have ? Answer: (9.63)(27) = 260 dollars
- 28. Suppose $\overline{x} = 13.9$ and $\sum x_i = 874$. Find *n* (which must be an integer). **Answer:** solve $\frac{874}{n} = 13.9$ to get n = 874/13.873 = 62.8777, so rounded we have n = 63.
- 29. Here are the Knoxville low temperatures recorded on Jan 1 1990-2013.

 $\{26,27,27,37,24,39,41,48,12,21,39,19,15,48,27,42,27,38,25,22,25,45,35,37\}$

Enter the data into L1 of your calculator and use 1-varstats to find the statistics.

(a) Find the sample size. **Answer:** n = 24 years from 1990 thru 2013 (b) Find $\sum x$. **Answer:** 746 (c) Find the mean. **Answer:** $\overline{x} = 31.08$

- (d) Find the median. Answer: 27
- (e) Find the range. Answer: 48 - 12 = 36

Distributions

30. Here is a sample of data for the number of siblings that math 201 students have.

 $\{1,0,2,1,5,2,0,1,3,2,1,0,0,2,4,3,0,3,1,1,1,2,4,0,1,2,2,3,1\}$

siblings	frequ	ency	re	lative freq	cumulative freq
0	6	<u>;</u>			
1	9)			
2	7	7			
3	4	ł			
4		2			
5]				
	siblings	freque	ncy	relative freq	cumulative freq
-	0	6		.207	.207
	1	9		.310	.517
Answor	2	7		.241	.758
Answer.	3	4		.138	.896
	4	2		.069	.965
	5	1		.034	1
-		29		1	

Fill out this frequency table:

31. Walking through a parking lot, an observer counts the number of cars by each manufacturer:

manufacturer	frequency
BMW (Euro)	2
Chrysler (Amer)	5
Ford (Amer)	9
GM (Amer)	13
Honda (Asia)	10
Kia (Asia)	4
Mercedes (Euro)	1
Nissan (Asia)	5
Subaru (Asia)	3
Toyota (Asia)	12
Volkswagen (Euro)	3

- (a) Is manufacturer a quantitative or qualitative random variable? Answer: qualitative
- (b) Fill out this frequency table, where the cars are grouped by the manufacturer's continent.

contine	ent	frequ	lency	re	elative freq
Americ	an				
Asiar	1				
Europe	ean				
		6	67		1
	\cos^{-1}	tinent	frequen	cy	relative freq
	Am	erican	27		.403
Answer:	A	sian	34		.507
	Eur	opean	6		.090
			67		1

- (c) Create a histogram for the continents' relative frequency.
- (d) Create a pie chart for the continents' relative frequency table.
- 32. This paper reports this following distribution for black bear litter size:

size	frequency	relative freq	cumulative freq
1	71		
2	206		
3	238		
4	74		
5	6		
	595		

(a) Fill in the relative and cumulative frequency columns.

	size	frequency	relative freq	cumulative freq
	1	71	.119	.119
	2	206	.346	.465
Answer:	3	238	.400	.865
	4	74	.124	.999
	5	6	.010	1
		595		

- (b) What percentage of births were twins? **Answer:** .346 = 34.6%
- (c) What percentage of births had 3 or fewer cubs? Answer: .865
- (d) What percentage of births had at least 3 cubs? Answer: 1-.465 = .535
- 33. A sample of n = 150 households yields this distribution for the number of pets owned.

pets	freq	relative	cumulative
0	41		.273
1	53		.627
2			.807
3	15		.907
4			.960
5	3		.980
6	2		.993
7	0		.993
8	1		1
	150		

Work backwards from the cumulative column to fill out the rest of the table. <u>pets</u> freq relative cumulative

	peus	neq	TCIAUIVC	cumulative
	0	41	.273	.273
	1	53	.353	.627
	2	27	.180	.807
	3	15	.100	.907
Answer:	4	8	.053	.960
	5	3	.020	.980
	6	2	.013	.993
	7	0	.000	.993
	8	1	.067	1
		150	1	

34. This table gives the distribution of U.S. public companies' market cap (in millions of dollars).

bin	frequency	PDF	CDF
0-100			.2202
100-500			.4964
500-1000			.6233
1000-5000			.8465
5000-10000			.9043
10000-100000			.9901
100000 +			1
	5655		

- (a) What percentage of the companies are under 1 billion dollars? (1 billion = 1000 million) Answer: 62.33~%
- (b) What percentage of the companies are over 1 billion dollars? Answer: 1 .6233 = .3767 = 37.67%
- (c) What percentage of the companies are between 1 billion and 10 billion dollars? Answer: .9043 .6233 = .281 = 28.1%

(d) Work backwards from the cumulative frequency column to fill out the relative frequency. You should use 4 digits of precision to avoid too much rounding error in this problem. Then, given that the sample size is n = 5655, fill out the frequency column (rounded to the nearest integer).

	bin	frequency	PDF	CDF
	0-100	1245	.2202	.2202
	100-500	1562	.2762	.4964
	500-1000	718	.1269	.6233
Answer:	1000-5000	1262	.2232	.8465
	5000-10000	327	.0578	.9043
	10000-100000	485	.0858	.9901
	100000 +	56	.0099	1
		5655	1	

35. This graph shows the distribution for cause of death in the U.S.



What percentage died from causes other than heart disease or cancer?

(a) in 1900

Answer: 1100 - 64 - 137.4 = 898.6 were from other causes, so 898.6/1100 = .817 = 81.7%(b) in 2010

Answer: 600 - 185.9 - 192.9 = 221.2 were from other causes, so 221.2/600 = .369 = 39.9%

36. Here are the points allowed by the 2014 Tennessee Titans:

10, 14, 16, 19, 21, 21, 26, 27, 27, 29, 30, 33, 36, 41, 43, 45

(a) Using these bins, fill out the distribution table.

points	freque	ency	rel	ative freq	cumulative free	ł
0-9						
10-19						
20-29						
30-39						
40-49						
	points	freque	ncy	relative freq	cumulative freq	
	0-9	0		0	0	
	10-19	4		.25	.25	
Answer:	20-29	6		.375	.625	
	30 - 39	3		.1875	.8125	
	40-49	3		.1875	1	
	-	16		1		

- (b) The Titans allowed less than 30 points in _____ percent of their games. Answer: 62.5~%
- (c) Draw a stem plot for the scoring data.

	stem	leaves
	1	0469
Answer:	2	116779
	3	036
	4	135

37. Which of these graphics does not represent a distribution?



Answer: Conversation topics - multiple responses were possible, so doesn't add up to 100%.38. For each random variable, sketch what you think the distribution would look like. Describe the shape.

- (a) Heights of 5 year old boys. Answer: symmetric, bell shaped
- (b) Population of U.S. cities. Answer: right-skewed
- (c) The number of free throws Lebron James would make in 10 attempts. Answer: left-skewed
- (d) Number of points scored in NBA basketball games. Answer: symmetric, bell-shaped
- (e) The number that would show up when you roll a fair die.Answer: uniform
- (f) Ages of people at a 5 year old's birthday party. Answer: bi-modal (kids and parents)
- 39. Consider this graph showing the evolving distribution of educational attainment in the U.S.



Population Age 25 and Over by Educational Attainment: 1940-2013

 $\label{eq:sources} {\tt SOURCES: www.census.gov/hhes/socdemo/education/data/cps/historical/index.html and www.oecdbetterlifeIndex.org/topics/education.}$

- (a) In 1940, what percentage of adults had a bachelor's degree or higher ? **Answer:** $\frac{3407}{3407+14627+56742} = 4.56\%$
- (b) In 2013, what percentage of adults had a bachelor's degree or higher ? **Answer:** $\frac{65506}{65506+116876+24517} = 31.7\%$
- 40. A Carson-Newman student has earned these course grades:

B,C,A,A,B,A,D,A,C,C,F,B,B,B,C,A,A,B,B,C,D,A,C,B,B

Let the random variable X be the GPA value of each grade.

(a) Fill in the distribution table.

grade	X	frequency			lative	cumulative	
F	0.0						
D	1.0						
С	2.0						
В	3.0						
А	4.0						
		2	5				
	grad	e X	frequer	icy	relative	cumulative	
	F	0.0	1		.04	.04	
	D	1.0	2		.08	.12	
Answer:	\mathbf{C}	2.0	6		.24	.36	
	В	3.0	9		.36	.72	
	Α	4.0	7		.28	1	
			25		1		

(b) Sketch a histogram. Is the distribution right skewed or left skewed. Answer: left

Location

41. This table shows the number of games played in NBA Finals' series from 1947-2014.

X	frequency
4	8
5	17
6	25
7	18

Find these statistics by hand.

- (a) Find the sample size. Answer: n = 68
- (b) Find the total number of games, $\sum x$. **Answer:** (4)(8) + (5)(17) + (6)(25) + (7)(18) = 393
- (c) Find the mean. **Answer:** $\overline{x} = \frac{393}{68} = 5.78$
- (d) Find the mode.Answer: 6 is the most frequent
- (e) Find the median. Answer: 6
- (f) Find the range. Answer: 7-4=3
- 42. Consider these distributions for men and women attactiveness ratings on a dating website:



- (a) Which gender has a higher mean? Answer: women
- (b) Which gender's distribution is more skewed? Left or right ? Answer: men, right
- 43. Create a list of n = 5 distinct test scores that satisfy the following:
 - median is 85 $\overline{x} = 80$ range is 35

Answer: one possible answer $\{60, 70, 85, 90, 95\}$

- 44. Here is a stem plot for the total points scored in Super Bowls I-XLIII.
 - 2 12379
 - 3 01116777899
 - 4 13445566777
 - 5 002345669
 - 6 115699
 - 7 5
 - (a) Is the distribution right skewed or left skewed? Answer: right-skewed
 - (b) Find the sample size. **Answer:** n = 43
 - (c) Given that $\sum x = 1954$, find the mean \overline{x} . **Answer:** $\overline{x} = \frac{\sum x}{n} = \frac{1954}{43} = 45.442$
 - (d) Find the mode(s). Answer: 31,37,47 (3 times each)
 - (e) Find the median without using your calculator. **Answer:** n = 43, so the 22nd value is 45
 - (f) Find the range. Answer: 75 - 21 = 54
 - (g) Find CDF(35), i.e. the percentage of games with ≤ 35 points scored. Answer: 9/43 = .209
- 45. Consider this frequency distribution for the viewer rating given to the movie "Office Space":

Х	frequency	7
1	2	
2	3	
3	5	
4	2	
5	8	
6	5	
7	9	
8	13	
9	23	
10	15	
(a)	sample size Answer:	n = 85
(b)	median Answer:	the 43rd value is
(c)	mode Answer:	9
(d)	mean Answer:	$\overline{x} = \frac{625}{85} = 7.35$
(e)	shape Answer:	left skewed

- 46. Go to the class data folder http://massey.limfinity.com/data/, and open the file "best-actress.xlsx" in Excel. Study the data, and the formula cells for the statistics.
 - (a) Enter the formula into Excel to find the median. **Answer:** = median(c2:c86) gives 33
 - (b) Find the range. **Answer:** 80 - 21 = 59
- 47. For practice with Excel, create a new file and enter similar data for the Best Actor award winners' ages.

http://en.wikipedia.org/wiki/List_of_Best_Actor_winners_by_age

On average, how much older is the best actor than the best actress ?

8

- 48. Suppose your assistant collected data on the weights of newborn babies. If her data set included a value of 912 pounds, speculate on the cause of this outlier.
 Answer: She probably mean 9 pounds 12 ounces. Extremely abnormal data should be checked for validity.
- 49. Here are the homeruns by season for Barry Bonds:

 $\{16, 25, 24, 19, 33, 25, 34, 46, 37, 33, 42, 40, 37, 34, 49, 73, 46, 45, 45, 5, 26, 28\}$

- (a) Enter the data in L1 on your calculator. Then calculate 1-Var-Stats L1.
- (b) Find Σx , the total number of homeruns in his career. Answer: $\Sigma x = 762$
- (c) How many seasons did he play? **Answer:** n = 22

- (d) Show your calculation to find the mean. Include the units. Answer: 762/22 = 34.64 HR/year
- (e) Find the median. Answer: 34
- (f) Find CDF(40). Answer: 15/22 = .682
- (g) Are there any "outliers"? Answer: yes, 5 and 73
- 50. If an economist is measuring the "average" income of workers in Knoxville, should she use the mean or the median? Explain.
 Answer: median. The distribution is probably right-skewed, with potentially large outliers. The median measures more of a "typical" income.
- 51. A family wants to know the "average" monthly electric bill over a two year span.
 - (a) What would be the sample size? Answer: n = 24
 - (b) Would you recommend the mean or the median? Answer: The mean, because the family is interested in the "total" spent, and a really expensive month should bring up the average.
- 52. Consider a dataset $\{7, 9, 12, 15\}$.
 - (a) Find the mean and median. Answer: mean 10.75, median 10.5
 - (b) Suppose one more observation gets appended to make {7,9,12,15,100}. Find the new mean and median.
 Answer: mean 35.75, median 12
- 53. A pizza delivery guy collected this data about his tip amounts.



- (a) For his sample size of n = 1031, his "average" (i.e. mean) tip amount was \$3.30. How many total dollars of tips did he collect ?
 Answer: (1031)(3.30) = 3402.3
- (b) Is the median tip amount more than 3.00? How do you know ? Answer: no, because more than half of the sample (84 + 228 + 288 = 600) gave ≤ 3.00 .
- 54. Here are hypothetical race finishes for two NASCAR drivers:
 - Tony Stewart: 1,3,7,12,35
 - Dale Earnhardt: 4,8,11,13,17
 - (a) Which driver has a better (lower) mean? Answer: Stewart 11.6, *Earnhardt 10.6
 - (b) Which driver has a better (lower) median? Answer: *Stewart 7, Earnhardt 11
 - (c) Which driver has an obvious outlier? Answer: Stewart's 35
 - (d) Which "average" gives more influence to the outlier? Answer: mean
- 55. It it possible to have a distribution for which 90% of the individuals are above average? If so, give an example. Answer for these two cases:
 - (a) if by "average" we intend the median.Answer: no, by definition 50% are above the median

- 56. Four roommates have a total of \$ 75 between them. The range is \$ 25, and the median is \$ 18. Also, the 2nd highest amount is 40% more than the 3rd highest. How many dollars does each roommate



- 57. Suppose the five number summary for the distribution of test grades is (34, 72, 83, 89, 98).
 - (a) Find the median. Answer: 83
 - (b) Find the range. **Answer:** 98 - 34 = 64
 - (c) Find the inter-quartile range. Answer: IQR 89 - 72 = 17
 - (d) Draw a boxplot.
 - (e) Describe the distribution's shape. Answer: left-skewed
- 58. Here are data for the distance from their home town for a sample of n = 29 Carson-Newman students.

 $\{1, 1, 2, 4, 5, 10, 12, 12, 20, 25, 25, 25, 30, 35, 50, 60, 75, 100, 120, 150, 150, 200, 210, 280, 300, 350, 400, 500, 750\}$

Find these statistics by hand.

(a) x

Answer: x

x

3902
29
134.55

(b) median

Answer: the 15th value is 50

(c) range

Answer: 750 - 1 = 749

(d) Q₁

Answer: the median of the first half is 12

(e) Q₃

Answer: the median of the second half is 205

(f) CDF(30)

Answer: 13/29 = .448

(g) Write the 5 number summary and sketch a boxplot. How would you describe the distribution's shape?

Answer: 1,12,50,205,750, right-skewed

- (h) Suppose you survey one more person, an international student that is 6000 miles from home. Compute the new mean and median. **Answer:** $\bar{x} = \frac{9902}{30} = 330.07$, and the new median is $\frac{50+60}{2} = 55$, so as expected, the median did not change nearly as much
- 59. Suppose that students take an IQ test and have the following scores:

 $\{116, 131, 98, 119, 104, 118, 100, 97, 111, 107, 112, 104, 142, 75, 126, 87, 120, 106, 109, 113, 123, 93, 105, 117\}$

- (a) Find the sample size n.Answer: 24
- (b) Is this data quantitative or qualitative? Answer: quantitative
- (c) (By hand) find the arithmetic mean. **Answer:** $\overline{x} = 109.71$
- (d) (By hand) find the median. Answer: 110
- (e) (By hand) find the range. **Answer:** 142 - 75 = 67
- (f) Enter the data in your calculator and check your answers.
- (g) Find the 5 number summary. Answer: 75, 102, 110, 118.5, 142
- (h) Find the inter-quartile range (IQR). **Answer:** $Q_3 - Q_1 = 16.5$
- (i) Sketch a histogram (use bins of 80-89, 90-99, 100-109, etc).
- (j) Create a stem plot of the data.
- (k) How would you describe the shape of this distribution?Answer: bell shaped
- (l) Are there any "outliers"? Answer: yes, 75 and 142
- 60. Suppose that for a symmetric distribution, $\overline{x} = 70$ and $Q_3 = 82$. Find Q_1 . Answer: 58
- 61. Create a list of n = 8 distinct numbers that satisfy the following conditions:
 - range is 80 median is 100 mean is 90 $\overline{x} < Q_1$

Answer: one possible answer: {25, 86, 97, 98, 102, 103, 104, 105}

62. Consider a random variable which records the number of working copy machines on campus on each school day. Suppose the five number summary is:

(9, 15, 18, 19, 20)

- (a) Find the range. Answer: 20 - 9 = 11
- (b) Find the inter-quartile range. **Answer:** 19 - 15 = 4
- (c) Draw a box plot. Is the distribution left skewed, right skewed, or symmetric? **Answer:** left-skewed, since Q_1 and the min are further from the median than Q_3 and the max.
- 63. Suppose a distribution has five number summary:

(4, 9, 11, 12, 14)

True or false? It must be true that the mean is $\overline{x} = 10$. Answer: false. The 5# summary is not the entire list of numbers in your data set, so it is incorrect to average the entries in the 5# summary. 64. Consider this distribution:

x	freq
2	54
3	31
5	20
7	12
11	7
13	4
17	3

Answer each question true or false. If false, find the correct value.

- (a) The sample size is n = 7. Answer: false, the sample size is the sum of the frequencies, which is n = 131
- (b) The median is x = 7. Answer: false, since n = 131, the median is the #66 in the list, which would be a 3
- (c) The mean is $\overline{x} = \frac{58}{7} \approx 8.29$. **Answer:** false, you have to take into account the frequencies to get $\overline{x} = \frac{565}{131} \approx 4.31$
- (d) The mode is 54. **Answer:** false, the *x*-value with the highest frequency is x = 2.
- 65. Find the mean litter size of black bears (see problem #3). Do it by hand, and then check your answer with 1-varstats.

size	frequency
1	71
2	206
3	238
4	74
5	6
	595

Answer: $\overline{x} = \frac{(1)(71) + (2)(206) + (3)(238) + (4)(74) + (5)(6)}{595} = 2.56$ 1-varstats L1,L2 gives $\overline{x} = 2.56$ cubs per litter

66. Suppose an investor has been accumulating shares of IBM stock. Here is a record of his purchases: date | shares | price per share

		1	1
1990	100		20
1995	120		23
2000	60		94
2005	80		76
2010	50		124

- (a) How many total shares did he purchase? Answer: 410
- (b) How much total money did he invest? Answer: 22680
- (c) Find the average price paid per share. **Answer:** $\overline{x} = 22680/410 = 55.32$ dollars per share
- (d) If IBM stock currently is worth \$145 per share, how much money has he made?
 Answer: current value is (145)(410) = 59450; minus amount paid gives 59450 22680 = 36770 dollars

67. The math department wants to find the mean GPA of its students.

class	# students	GPA
algebra	90	3.15
stats	50	3.0
calculus	20	2.6

Find the mean weighted by class size. Answer: 3.034 (be sure L1 is the GPA and L2 is the frequency)

68. Here is a student's course grade distribution. Assume all courses are worth the same amount of credit.

grade	X	frequency	relative	cumulative
F	0.0	1	.04	.04
D	1.0	2	.08	.12
\mathbf{C}	2.0	6	.24	.36
В	3.0	9	.36	.72
А	4.0	7	.28	1
		25	1	

- (a) Show how you would calculate \overline{x} (the student's GPA) by hand. **Answer:** $\overline{x} = \frac{0(1)+1(2)+2(6)+3(9)+4(7)}{25} = 69/25 = 2.76$
- (b) What is the mode of the grade distribution? **Answer:** "B" or X = 3 has the highest frequency
- 69. A forest ranger has estimated the heights of trees growing in a certain grove:

height (feet)	count
0-10	25
10-20	13
20-30	10
30-40	8
40-50	9
50-60	12
60-70	18
70-80	7
80-90	2

(a) Enter the bin midpoints as L1 in your calculator, and the count column in L2. Find the mean height.

Answer: 1-varstats L1,L2 gives $\overline{x} = 36.3$

- (b) Find the mean height of only the "mature" trees that are at least 50 feet tall. **Answer:** using only the last 4 bins, you get $\overline{x} = 64.7$
- 70. An economist wants to aggregate estimates for the monthly new jobs number. There are 5 reports, each getting a different weight based on its track record.

report	weight
190	9
220	7
140	7
170	5
240	2

(a) Find the unweighted mean of the five reports. **Answer:** 1-varstats L1 gives $\overline{x} = 192$ (b) Find the weighted mean. **Answer:** 1-varstats L1,L2 gives $\overline{x} = 185.3$

children $(L1)$	households (L2)	# kids (L3)
1	20	20
2	43	86
3	24	72
4	9	36
5	3	15
6	1	6
	100	235

71. Consider this information for a particular school district:



- (a) How many children? Answer: 235
- (b) How many households (sample size)? Answer: 100
- (c) Find the average children per household. **Answer:** $\frac{235}{100} = 2.35$ (or use 1-varstats L1,L2)
- (d) Household Size Paradox: What if you polled the school children, asking each one how many children lived in their household? If you weight the responses by the number of kids instead of the number of households, what value would you get for the mean?
 Answer: 1-varstats L1,L3 gives x = 2.82, this higher number occurs because larger families

have more kids to respond

- 72. The 75th percentile is the 3rd quartile.
 The 80th percentile is the _____ quintile, and also the _____ decile.
 Answer: 4th quintile, 8th decile
- 73. Here is the histogram and the distribution table for the U.S. population ages as of 2009.



A	В	С	D	E	F	G	Н	I	J	К
age in 2009	thousands	pdf	cdf			age	thousands	pdf	cdf	
0	4261	0.0138803	0.0138803	0		51	4456	0.0145134	0.7159332	227249
ĩ	4298	0.0139994	0.0278797	4298		52	4397	0.0143207	0 7302539	228628
2	4336	0.0141224	0.0420022	8672		53	4218	0.0137402	0 7439941	223579
3	4224	0.0137569	0.0557590	12671		54	4230	0.0137786	0 7577727	228435
4	4181	0.0136171	0.0693762	16723		55	4040	0.0131603	0.7709330	220433
	4101	0.0136330	0.0033702	20029		56	2000	0.0131003	0.7936394	219270
6	/130	0.013/1812	0.0050052	20520		57	3750	0.0120334	0.7050204	210270
7	4109	0.0133012	0.0904904	24034		59	3652	0.0122441	0.7550724	211799
0	4100	0.0135012	0.1030/17	20730		50	2626	0.0110330	0.8077000	211709
0	4107	0.0130731	0.1254440	26097		59	3020	0.0110112	0.0193772	213340
9	4010	0.0130600	0.1303040	30007		61	2479	0.0113327	0.0309099	200700
10	3940	0.0120341	0.1495590	39404		62	3430	0.0111975	0.0421072	209705
11	3941	0.0128300	0.1021950	43332		62	3087	0.0110835	0.853/90/	222390
12	3937	0.0128873	0.1750829	4/4/9		03	2000	0.0086845	0.8024752	16/9/6
13	4033	0.01313/1	0.1882201	52433		64	2042	0.0086038	0.8/10/90	169050
14	4096	0.0133417	0.2015017	5/340		C0	2088	0.0084292	0.8795082	108213
15	4134	0.0134642	0.2150259	62006	1	00	2000	0.0086509	0.8881590	1/5293
16	4225	0.0137609	0.2287869	6/59/		6/	2329	0.0075859	0.8957449	156042
1/	4307	0.0140278	0.2428147	/3215		68	2145	0.0069865	0.9027314	145858
18	4389	0.0142944	0.2571091	78995		69	2067	0.0067309	0.9094623	142589
19	4484	0.0146046	0.2/1/13/	85193		/0	1949	0.0063469	0.9158092	136403
20	4340	0.0141361	0.2858498	86800		/1	1893	0.0061647	0.9219740	134380
21	4291	0.0139777	0.2998274	90119		12	1/65	0.0057505	0.9277244	12/116
22	4266	0.0138947	0.3137222	93850		/3	1/12	0.0055748	0.9332992	124943
23	4306	0.0140269	0.3277490	99049		74	1689	0.0055026	0.9388019	125016
24	4336	0.0141222	0.3418713	104058		75	1529	0.0049793	0.9437811	114654
25	4264	0.0138883	0.3557596	106599		76	1506	0.0049062	0.9486873	114478
26	4330	0.0141040	0.3698635	112584		77	1463	0.0047639	0.9534512	112620
27	4350	0.0141692	0.3840327	117455		78	1422	0.0046300	0.9580813	110877
28	4380	0.0142662	0.3982989	122639		79	1406	0.0045809	0.9626622	111108
29	4353	0.0141799	0.4124788	126251		80	1295	0.0042177	0.9668799	103593
30	4136	0.0134721	0.4259509	124085		81	1249	0.0040674	0.9709473	101150
31	4013	0.0130714	0.4390223	124407		82	1173	0.0038194	0.9747667	96155
32	3950	0.0128665	0.4518888	126408		83	1083	0.0035268	0.9782935	89871
33	3844	0.0125207	0.4644095	126855		84	1023	0.0033329	0.9816264	85953
34	3945	0.0128495	0.4772590	134130		85	930	0.0030291	0.9846556	79050
35	3824	0.0124557	0.4897147	133844		86	840	0.0027360	0.9873916	72240
36	3909	0.0127328	0.5024475	140731		87	720	0.0023451	0.9897367	62640
37	4093	0.0133317	0.5157793	151444		88	640	0.0020846	0.9918213	56320
38	4316	0.0140579	0.5298372	164008		89	560	0.0018240	0.9936453	49840
39	4396	0.0143183	0.5441555	171443		90	470	0.0015309	0.9951762	42300
40	4156	0.0135360	0.5576915	166231		91	380	0.0012377	0.9964139	34580
41	4077	0.0132787	0.5709702	167149		92	290	0.0009446	0.9973585	26680
42	4084	0.0133033	0.5842736	171543		93	210	0.0006840	0.9980425	19530
43	4196	0.0136673	0.5979408	180432		94	150	0.0004886	0.9985310	14100
44	4479	0.0145875	0.6125283	197058		95	125	0.0004071	0.9989382	11875
45	4543	0.0147976	0.6273259	204440		96	100	0.0003257	0.9992639	9600
46	4524	0.0147353	0.6420613	208104		97	75	0.0002443	0.9995082	7275
47	4535	0.0147711	0.6568323	213144		98	55	0.0001791	0.9996873	5390
48	4576	0.0149050	0.6717374	219653		99	40	0.0001303	0.9998176	3960
49	4653	0.0151552	0.6868926	227992		100	25	0.0000814	0.9998990	2500
50	4460	0.0145272	0.7014197	223004		101	15	0.0000489	0.9999479	1515
						102	8	0.0000261	0.9999739	816
						103	5	0.0000163	0.9999902	515
						104	3	0.0000098	1.0000000	312
						totals	307017	1.0000000		11372531

- (a) Describe the shape of this distribution. Answer: right-skewed
- (b) Find the mode. Answer: the highest frequency was for 49 year olds
- (c) Find the mean. **Answer:** $\overline{x} = \frac{11372531}{307017} = 37.04$
- (d) Find the median. Answer: CDF passes .50 at 36
- (e) Find the IQR. **Answer:** $Q_3 - Q_1 = 54 - 18 = 36$
- (f) What percentage of the population is ≤ 16 years old. Answer: CDF(16) is .229 = 22.9%
- (g) What percentage of the population is at least 62 years old. Answer: not ≤ 61 , so 1 - .842 = .158 = 15.8%

- (h) What percentage of the population is in their 20's (20-29) ? **Answer:** $CDF(29) - CDF(19) \approx .412 - .272 = .14 = 14\%$
- (i) Find the 20th percentile age. **Answer:** look up .2 in the CDF column to see x = 14
- (j) Find the 90th percentile age. **Answer:** look up .9 in the CDF column to see x = 68
- (k) A 52 year old person is at the _____ percentile. **Answer:** cdf(52) = .73, so the 73rd percentile
- 74. Open http://massey.limfinity.com/data/fleet.txt in Excel. You can either cut/paste or use the "data-from web" Excel tool. Create PDF and CDF columns, and do a histogram. It should look like this:



This lists a frequency distribution of the age (in months) of a rental car company's fleet.

- (a) Find the sample size n by summing the frequency column. Answer: n = 1879
- (b) Find the mode.Answer: 9 months has the highest frequency
- (c) Find the 5 number summary (using the CDF column to get the quartiles).Answer: 0,7,13,21,36

- (d) Find the range. Answer: 36 - 0 = 36
- (e) Find the IQR. Answer: 21 - 7 = 14
- (f) Use Excel to find the mean age. **Answer:** $\overline{x} = \frac{26764}{1879} = 14.24$ months
- (g) What percentage of the fleet is 24 months old? **Answer:** PDF of x = 24 is .0218 = 2.18%
- (h) What percentage of the fleet is 24 or fewer months old? **Answer:** CDF of x = 24 is 83.77%
- (i) What percentage of the fleet is at least 24 months old?
 Answer: 1 (CDF of x = 23) = 1 .8159 = .1841 = 18.41%
- (j) The 22nd percentile is _____ months. Answer: 6
- (k) 22 months is the _____ percentile. Answer: 79th
- 75. Open http://massey.limfinity.com/data/knox2010.xlsx in Excel. These are the daily temperatures recorded in Knoxville in 2010.
 - (a) Highlight the "high" column, and use a pivot table/chart to create the frequency distribution.
 - (b) Create PDF and CDF columns for your frequency table.
 - (c) Find the 5 number summary (using the CDF column to get the quartiles).
 - (d) Use Excel to find the mean temperature.
 - (e) What percentage of the days were recorded as 70 degrees? **Answer:** PDF of x = 70
 - (f) What percentage of the days were ≤ 40 degrees? Answer: CDF of x = 40
 - (g) What percentage of the days at least 70 degrees? **Answer:** 1 - (CDF of x = 69)
- 76. Consider this growth chart for girls age 2-20.



(a) On her 5th birthday, a girl is 45 inches tall. Find her percentile, and follow the curve to estimate her height at age 20.

Answer: roughly the 90th percentile, which ends at around 68 inches.

(b) On her 10th birthday, a girl weighs 60 pounds. Find her percentile, and follow the curve to

estimate her weight at age 20. Answer: roughly the 25th percentile, which ends at around 115 pounds.

- 77. You have a sample of data with n = 12 and $\overline{x} = 50$. Your research partner has a sample of data with n = 5 and $\overline{x} = 60$. If you combine your results, find the new sample size and mean. **Answer:** n = 17 and $\overline{x} = \frac{(12)(50)+(5)(60)}{17} = 52.9$
- 78. Here is the distribution of tenure (year of employment) for workers at a startup tech company.

Х	PDF	CDF
1		.24
2		.39
3		.73
4		.85
5		.94
6		1

- (a) Fill out the PDF column.
- (b) Find the mean. **Answer:** (1)(.24) + (2)(.15) + (3)(.34) + (4)(.12) + (5)(.09) + (6)(.06) = 2.85
- (c) Check that the mean equals the last X value (6) minus all but the last CDF values,
 i.e. x̄ = 6 .24 .39 .73 .85 .94.
 Bonus points to the first two students that come to me with a good explanation of why that trick works.

Geometric Mean

- 79. Consider the data $\{1, 10, 100, 1000\}$.
 - (a) Find the arithmetic mean. Answer: (1 + 10 + 100 + 1000)/4 = 277.75
 - (b) Find the geometric mean. **Answer:** $(1 \cdot 10 \cdot 100 \cdot 1000)^{1/4} = 31.6$
 - (c) Find the median. **Answer:** (10 + 100)/2 = 55
- 80. A publication ranks colleges by several factors: selectivity, cost, teaching, campus, technology. Find the arithmetic mean \overline{x} , and the geometric mean \overline{x}_g for these two schools' rankings within the South Atlantic Conference.
 - (a) Karsten-Neuman: $\{3, 3, 1, 3, 10\}$ **Answer:** $\overline{x} = \frac{20}{5} = 4$ and $\overline{x}_g = (3 \cdot 3 \cdot 1 \cdot 3 \cdot 10)^{1/5} = 3.06$
 - (b) Tuskuphlegm: $\{4, 4, 4, 4, 4\}$ **Answer:** $\overline{x} = \frac{20}{5} = 4$ and $\overline{x}_g = (4 \cdot 4 \cdot 4 \cdot 4 \cdot 4)^{1/5} = 4$

Why do you think the geometric mean favors K-N ? Answer: K-N beats Tusk in 4 out of 5 categories, and the geometric mean does not penalize the #10 ranking as much.

81. Sales at your business have gone up by 30%, down by 10%, and up by 20% in the last three years respectively. Use the geometric mean to find the annual percentage gain.
Answer: ³√(1.3)(.9)(1.2)) = 1.1197, so the annual gain is about 11.97 %

82. An employee received the following percentage raises over the last 6 years.

2, 7, 20, 3, 0, 5

- (a) Use the geometric mean to compute her average annual raise. **Answer:** $((1.02)(1.07)(1.20)(1.03)(1)(1.05))^{1/6} = 1.0597$, so this equates to about 5.97% annually
- (b) If her starting salary was 35,000, then what is her current salary? Answer: $35000(1.0597)^6 = 49575$
- 83. Yale University tuition/room/board has gone from \$6425 in 1976 to \$49800 in 2010. Find the average annual percentage increase during that time.
 Answer: (49800/6425)^{1/34} = 1.062, so 6.2% annually
- 84. The population of Virginia has gone from 750 thousand in 1790, to 7.9 million in 2010. Find the average annual population growth rate.

Answer: $\left(\frac{7,900,000}{750,000}\right)^{1/220} = 1.01076$, which indicates 1.076% annual growth

- 85. One dollar invested in the U.S. stock market in January 1871 would have been worth \$161,302 in January 2012. Find the average annual return. (these numbers include dividends).
 Answer: (161302/1)^{1/141} = 1.089, so 8.9% annual return
- 86. The number of cases of a disease has decreased from 5000 in 1990 to 3000 in 2010. On average, this is a _____ percent annual decrease.
 Answer: (3000/5000)^{1/20} = .975, so 2.5% annual decrease
- 87. Suppose a new car loses half of its original value after four years. Find the average annual depreciation. **Answer:** $(1/2)^{1/4} = .84$, so it loses about 16% of its value annually

Dispersion

88. As of Jan 1, 2011, the U.S. Supreme court justice ages were:

 $\{62, 74, 72, 55, 60, 77, 74, 50, 56\}$

Use your calculator to find the standard deviation. Include units. Answer: 1-varstats L1 gives s = 9.95 years

89. Find the standard deviation of black bear litter size (see problem #3). Include units.

size	frequency
1	71
2	206
3	238
4	74
5	6
	595

Answer: 1-varstats L1,L2 gives s = .892 cubs

- 90. Visit http://www.imdb.com/title/tt0499549/ratings and study the distribution of movie ratings for "Avatar".
 - (a) Describe the shape.

Answer: left-skewed (notice that IMDB has the ratings 10-1 instead of 1-10)

- (b) Use your calculator to find the mean and standard deviation.
- 91. Suppose the standard deviation is 7, then what is the variance? Answer: the variance is $s^2 = 49$
- 92. The variance in the total points scored in a college football game is 225. Find the standard deviation. Answer: $s^2 = 225$, so $s = \sqrt{225} = 15$ points
- 93. Suppose the Associated Press college football poll is comprised of 67 ballots. The rankings for a particular team are given by this frequency table.

rank	freq	PDF	CDF
1	3	.045	.045
2	9	.134	.179
3	27	.403	.582
4			
5	7		.910
6			.970
7	1	.015	.985
8	1	.015	1
	67	1	

(a) Fill out the relative frequency (PDF) and cumulative frequency (CDF) columns.

	rank	freq	PDF	CDF
	1	3	.045	.045
	2	9	.134	.179
	3	27	.403	.582
Answer:	4	15	.224	.806
	5	7	.104	.910
	6	4	.060	.970
	7	1	.015	.985
	8	1	.015	1

(b) Find the median without using a calculator.

Answer: n = 67, so the 34th value is the median, which is x = 3. Or note the CDF passes .5 when x = 3

- (c) Explain why $Q_1 = Q_2$ in this distribution. **Answer:** The median, $Q_2 = 3$ from previous answer. $Q_1 = 3$ as well since the CDF passes .25 when x = 3.
- (d) Use your calculator to find the mean and standard deviation. **Answer:** $\overline{x} = 3.52, s = 1.36$
- (e) Create a histogram of the data.
- (f) Describe the shape of the data. Answer: right skewed
- 94. Which would you expect to have a larger standard deviation in ages: professional football players, or professional statisticians?

Answer: statisticians, since they may be anywhere say from 22 to 65, but football player ages are in a much tighter range say 22 to 35

- 95. Assuming the means are identical, which would you expect to have a lower standard deviation: the weekly revenue of an auto dealer or the weekly revenue of a grocery store.Answer: the grocery store, since grocery spending is more regular and consistent
- 96. Suppose two horses are training for the Kentucky Derby. In practice runs, the horses have the same average performance. But Horse A has a higher variance than Horse B. If both horses are long shots, which would you bet on to win the race? Explain.
 Answer: Bet on Horse A, because it is less predictable, so it has a better chance of having an extraordinary run.
- 97. Suppose stock ABC has a 80% higher variance of daily returns than stock XYZ. Then ABC has ______ percent high standard deviation than XYZ. Answer: $s_1^2 = 1.8s_2^2$, so $s_1 = \sqrt{1.8s_2} = 1.34s_2$, so 34% higher
- 98. Among the general population, a person can hold his/her breath under water for 45 seconds on average. The standard deviation is 32 seconds.
 - (a) What shape would you expect this distribution to have?Answer: right-skewed, since nobody would be below 0 and some may be pretty high
 - (b) Find the z-score of a stunt man that can hold his breath for 7 minutes. **Answer:** $z = \frac{420-45}{32} = 11.72$
- 99. Compare z-scores to decide which would be more "unusual".
 - annual rainfall of 30 inches in a location with $\overline{x} = 40$ and s = 12
 - annual rainfall of 20 inches in a location with $\overline{x} = 12$ and s = 6

Answer: the z-scores are $\frac{30-40}{12} = -.83$ and $\frac{20-12}{6} = 1.33$ respectively, so the latter one is more unusual

- 100. If x = 60 has a z-score of 0.75 from a distribution with standard deviation 12, then what is \overline{x} ? **Answer:** solve the equation $\frac{60-\overline{x}}{12} = .75$ to get $\overline{x} = 51$
- 101. Suppose that a psychologist uses the HFU (happy feeling units) scale to measure her patients' moods. The mean on this scale is 500, and the standard deviation is 200. There is another mood measurement called the Mood Index, which is centered at 100 with a standard deviation of 25. Use z-scores to convert 350 HFU to a Mood Index value. **Answer:** $z = \frac{350-500}{200} = -.75$, and then solve $-.75 = \frac{x-100}{25}$ to get x = 81.25
- 102. A teacher wants to see how her class is doing, so she observes recent test scores:

 $\{91, 83, 37, 99, 85, 89, 59, 78, 75, 65, 89, 99, 95, 63, 74, 87, 52, 83, 92, 75, 81\}$

- (a) Use your calculator to find the mean, median, and standard deviation. **Answer:** 1-varstats L1 gives $\overline{x} = 78.62$, median = 83, s = 15.98
- (b) Find the range. **Answer:** 99 - 37 = 62
- (c) Find the inter-quartile range.Answer: from your calculator Q3-Q1
- (d) Sketch a boxplot.
- (e) What is the z-score of a grade of 70 ? **Answer:** $z = \frac{70-78.62}{15.98} = -.54$

- (f) Make a stem plot.
- (g) Describe the shape of this data distribution.Answer: left skewed (notice that the mean is less than the median)
- 103. Suppose a bell-shaped distribution has $\overline{x} = 20$, standard deviation s = 6, and five number summary 4, 16, 20, 24, 36.
 - (a) What is the z-score of x = 23 ? **Answer:** $z = \frac{23-20}{6} = 0.5$
 - (b) What measurement corresponds to z = -1.33 ? Answer: solve $-1.33 = \frac{x-20}{6}$ to get x = 12
 - (c) The middle 50% of the data lie between _____ and _____. Answer: 16 and 24

Missing Values

104. Here are sample data of Netflix user ratings for the movie "Mean Girls".

stars	freq	PDF	CDF
1			
2	752	.047	.072
3			.293
4	7024		
5		.268	
	n		

- (a) Find the sample size n. **Answer:** solve $\frac{752}{n} = .047$ to get n = 16000
- (b) Fill in the missing table entries.

	stars	freq	PDF	CDF
	1	400	.025	.025
	2	752	.047	.072
Answer:	3	3536	.221	.293
	4	7024	.439	.732
	5	4288	.268	1
		16000	1	

- (c) Find the mean rating. **Answer:** $\overline{x} = 3.878$
- (d) Find the median rating. **Answer:** CDF passes .5 when x = 4 stars
- 105. An all-you-can-eat pizza buffet has studied the behaviour of its customers and found the following distribution for the number of large slices eaten.

x	freq	PDF (relative freq)	CDF (cumulative freq)
1	24		
2	39		.315
3	52		.575
4	33		
5			.845
6	16		
7	11		
8	4		1
	n	1	

- (a) Find the relative frequency for x = 3. Answer: .575 - .315 = .26
- (b) Find the sample size n. **Answer:** solve $\frac{52}{n} = .26$ to get n = 200
- (c) Complete the table.

	x	freq	PDF	CDF
	1	24	.120	.120
	2	39	.195	.315
	3	52	.260	.575
Answor	4	33	.165	.740
Answer.	5	21	.105	.845
	6	16	.080	.925
	7	11	.055	.980
	8	4	.020	1
		200	1	

- (d) Find \overline{x} by hand. What are the units? **Answer:** $\overline{x} = \frac{24(1)+39(2)+52(3)+33(4)+21(5)+16(6)+11(7)+4(8)}{200} = \frac{700}{200} = 3.5$ slices per customer.
- (e) Find the 5 number summary (use the CDF column to find the quartiles).Answer: 1,2,3,5,8
- (f) Describe the shape. Answer: right skewed
- 106. Use information in this table to find the sample size.

freq	PDF	CDF
150		
		.62
100		
60		1
\overline{n}	1	

Answer: note that 1 - .62 = .38 is in the last two rows, so 160 = .38n and n = 421

x	freq	PDF	CDF
0			
1	3		.5
2			
3	8	.364	1
	n		

107. Consider the partially completed distribution table:

(a) Determine the value of n, and then complete the rest of the table.

	x	freq	PDF	CDF
	0	8	.364	.364
Answor	1	3	.136	.5
Answer:	2	3	.136	.636
	3	8	.364	1
		n		

- (b) Sketch a histogram and describe the shape. **Answer:** symmetric
- 108. Suppose Alex, Brad, Carl, and Dan have a mean of \$45 in cash. Earl shows up, and the mean amount of cash for the five guys rises to \$63. How much cash does Earl have? **Answer:** the total cash is (63)(5) = 315, and the first four guys had (45)(4) = 180, so Earl must have had \$135
- 109. The offensive line of a football team has a center at 305 pounds, left guard at 290, right guard at 297, and left tackle at 310. If the average weight of the five offensive linemen is 302 pounds, how much does the right tackle weigh? **Answer:** solve $\frac{305+290+297+310+x}{5} = 302$ to get x = 308

- 110. In the first three weeks of a diet, Marge lost 45, 12, and 33 ounces respectively. How many ounces does she need to lose in the 4th week to reach her weight loss goal of 2 pounds per week? (a pound is 16 ounces) **Answer:** solve $\frac{45+12+33+x}{4} = 32$ to get x = 38 ounces
- 111. Eight students took a test, and the average grade was 82. Priscilla was sick that day, and took a make-up test. Her grade brought the class average down to an 80. What was her grade? **Answer:** solve $\frac{(8)(82)+(1)(x)}{9} = 80$ to get x = 64
- 112. An investor purchased 20 shares of Apple at \$250 per share. Later she bought 30 more shares. The average price paid for all the shares was \$290.20. What was the average price paid for the last 30 shares?

Answer: solve $290.2 = \frac{20(250)+30x}{50}$ to get x = 317

- 113. Suppose that in your history class, you have a 85 average on your tests, a 92 average on your term papers, and a 60 average on quizzes. These components make up 40%, 25%, and 10% of your final grade respectively. If your final exam is 25% of your final grade:
 - (a) What is your grade entering the exam? **Answer:** ((.40)(85) + (.25)(92) + (.10)(60))/(.75) = 84
 - (b) What is your final grade if you get a 80 on the exam? **Answer:** (.40)(85) + (.25)(92) + (.10)(60) + (.25)(80) = 83

- (c) What is the minimum final exam grade you must get to receive a B in the class? **Answer:** solve (.40)(85) + (.25)(92) + (.10)(60) + (.25)(x) = 79.5 to get x = 66
- (d) Is it possible to get an A in the class? **Answer:** (.40)(85) + (.25)(92) + (.10)(60) + (.25)(x) = 89.5 implies you need x = 106, so you can't get an A unless you get bonus/extra credit
- 114. Two hundred elderly people were surveyed to determine how many times they had been married.

# marriages	frequency
0	y
1	114
2	x
3	14
4	3

If the mean is 1.4 marriages per person, find the missing entries. **Answer:** solve $\frac{(0)(y)+(1)(114)+(2)(x)+(3)(14)+(4)(3)}{200} = 1.4$ to get x = 56, and so y = 13.

115. Here is information about cupcake consumption at a birthday party:

- Four people didn't eat any cupcakes.
- Eleven people ate 1 cupcake.
- Seven people ate 3 cupcakes.
- Everyone else ate 2 cupcakes.
- The mean was 1.75 cupcakes per person.

How many people were at the party? (Hint: first solve for the number of people that ate 2 cupcakes.) **Answer:** solve $1.75 = \frac{(0)(4)+(1)(11)+(2)(x)+(3)(7)}{4+11+7+x}$ to get x = 26, so there were 48 people at the party.

116. Here are the hits for a rec-league softball player:

	bases	freq
out	0	x
single	1	75
double	2	41
triple	3	6
home run	4	y
		200

Her slugging percentage is $\overline{x} = 1.215$ bases per at-bat. Solve for x and y. **Answer:** solve $\frac{(0)(x)+(1)(75)+(2)(41)+(3)(6)+(4)(y)}{200} = 1.215$ to get y = 17, so then x = 61

- 117. Suppose there is a distribution such that
 - when x = 19, z = 0.25
 - when x = 9, z = -1

Find \overline{x} and s.

Answer: We have two equations $\frac{19-\overline{x}}{s} = .25$ and $\frac{9-\overline{x}}{s} = -1$. Rearrange to get $19 - \overline{x} = .25s$, $9 - \overline{x} = -s$, so $\overline{x} = 19 - .25s$ and $\overline{x} = 9 + s$. Substitute to get 19 - .25s = 9 + s, so 10 = 1.25s and s = 8. Then $\overline{x} = 9 + 8 = 17$



Statistics in Perspective

- 118. Every year, the U.S. gives approximately \$ 50 billion in foreign aid. Per-capita, that equals about dollars per U.S. citizen. $\frac{50,000,000,000}{300,000,000} = 167$ Answer:
- 119. About 2.3 million people die of AIDS each year. That is about one every ____ seconds. **Answer:** 31536000/2300000 = 13.7
- 120. In 2013, China had a population 1.35 billion, and a GDP of \$9.19 trillion. What was the Chinese per-capita GDP ? $\frac{9,190,000,000,000}{1,350,000,000} = 6807$ Answer:
- 121. Suppose you drove 100 miles at 50 MPH, then drove 210 miles at 70 MPH. Find your average speed for that trip. **Answer:** The first part of the trip takes 2 hours, and the second part takes 3 hours. So you went a total of 310 miles in 5 hours. The average speed is $\frac{310}{5} = 62$ MPH.
- 122. At a certain college, 70% of female applicants were admitted, and 60% of male applicants were admitted. Is it necessarily true that 65% of all applicants were admitted? **Answer:** not unless there were an equal number of male and female applicants
- 123. In a recent year, Jefferson County residents spent 8.4 million dollars on state lottery tickets. If the per-capita spending was \$165, then what is the population of Jefferson Co ? Answer: solve $\frac{8,400,000}{n} = 165$ to get n = 50909

Probability and Expected Value

- 124. Suppose that when a student parks in a faculty parking spot, she has a 23% chance of getting a ticket. What is the probability that she does not get a ticket? **Answer:** 1 - .23 = .77 = 77%
- 125. If Carson-Newman's soccer team has a 53% chance of winning and a 29% chance of losing, then what is the probability that the game ends in a tie? **Answer:** 1 - .53 - .29 = .18 = 18%
- 126. Let the random variable X be a student's grade on a statistic test. If $P(X \le 79) = .37$, then what is P(X > 79) ? **Answer:** 1 - .37 = .63
- 127. Is this an example of (primarily) empirical, subjective, or theoretical probability?
 - (a) There is a 12% chance that the Lady Vols will win the national championship. **Answer:** subjective
 - (b) There is a 25% chance of drawing a spade from a well shuffled deck of cards. **Answer:** theoretical
 - (c) The stock market has gone up 75 of the last 100 years, so there is a 75% chance it will go up in a given year.

Answer: empirical

(d) The stock market has a 95% chance of going up this year because the president will fix the economy.

Answer: subjective