Ratios, Proportions, and Percentages

Each of you must bring a gift in **proportion** to the way the Lord your God has blessed you.
– Deuteronomy 16:17

Instructions

- Read everything carefully, and follow all instructions.
- Do the numbered problems that are boxed. Show your work neatly in the allotted space.
- To receive full credit, you must justify your answer by showing your work or calculator commands.
- Circle your final answer, or write it in the spot provided.
- You may work with others, or ask for help. Your answers should reflect your own understanding of the material.
- Select, unspecified, parts of this take-home project may be graded to determine ☐ % of your grade.
- Neatly tear the pages out of your book, and have them prepared to submit in class on the due date.
- On the due date, there will be a short in-class portion to determine ☐ % of your grade.

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1 Ratios

A **ratio** is the quotient relating one quantity to another.

(EXAMPLE) A basketball team has made 18 shots and missed 12. The ratio of made to missed is

$$\frac{18}{12} = \frac{3}{2} = \frac{1.5}{1}$$

- A ratio may be written using a colon. The made to missed ratio is 3 : 2, read "three to two".
- The reduced ratio 1.5 : 1 shows that the team made 1.5 times as many shots as they missed.

1. (4 pts) At a certain engineering school, 209 graduates were male and 76 were female. As reduced integers, the male to female ratio is " ☐ to 4", which further reduces to " ☐ to one".

   Answer: $\frac{209}{76} = \frac{11}{4} = 2.75$, so "11 to 4" and "2.75 to one"

2 Ratios and Rates

Some people make a distinction between a “rate” and a “ratio”.

- A “ratio” relates two things having the same units, so the units cancel out, e.g.

$$\frac{18 \text{ shots}}{12 \text{ shots}} = 1.5$$

- A “rate” relates two things having different units, e.g.

$$\frac{130 \text{ miles}}{2 \text{ hours}} = 65 \text{ miles per hour (MPH)}$$

This project is not pedantic about keeping this distinction.

2. (3 pts) Suppose Apple's stock price is $P = \$500$ per share, and the company earns $E = \$46$ per share. Investors refer to the value P/E as the “price to earnings ratio”. Compute Apple's P/E ratio.
   Answer: $\frac{500}{46} = 10.87$

3 Using Ratios

Using a ratio, you can solve for unknown quantities.

- (EXAMPLE) Suppose the Euro-Dollar exchange rate is 10:13. That means 10 euros can be exchanged for 13 dollars. After a trip to Europe, you have 75 euros left over. How many dollars will they convert to?

  Let x be the unknown number of dollars, and equate the ratios:

$$\frac{10}{13} = \frac{75}{x}$$

  Solve for x to get $10x = (13)(75)$ so $x = \frac{975}{10} = 97.5$, so you would get \$97.50.

- (EXAMPLE) A milkshake stand sells chocolate and vanilla shakes. This summer they sold a total of 22178 shakes, and the chocolate to vanilla ratio was $19 : 7$. How many were chocolate?

  The easiest way to solve this problem is to notice that $\frac{19}{26}$ of the shakes are chocolate. This fraction of the total is $\frac{19}{26}(22178) = 16207$ chocolate shakes.

3. (4 pts) The yea : nay ratio for voting on a particular bill in the U.S. House of Representatives was $7 : 5$. A total of 432 votes were cast. How many were 'yea' and how many were 'nay'?
   Answer: We know that for every 7 yea votes there were 5 nay votes, so $\frac{7}{12}$ of the votes were yay. Let x be the number of yea votes. Solve $\frac{7}{12} = \frac{x}{432}$ to get $x = 252$ yea votes, and 180 nay.

4 Proportions

A **proportion** is a ratio of the form part : whole.

- The numerator is the size of the part, and the denominator is the size of the whole.
- In the discrete case, **relative frequency** is the same as proportion, denoted by the letter "p".

$$p = \frac{x}{n}$$

- Since the part can't be bigger than the whole, a proportion must be between 0 and 1.

(EXAMPLE) The basketball team made 18 shots and missed 12. The "whole" is the 30 shots that they attempted. The proportion of made shots is 18:30, or

$$p = \frac{18}{30} = \frac{3}{5} = 0.60$$

4. (3 pts) The proportion of students that graduated in four years is $p = .45$. If a freshman class had 500 students, how many graduated in four years?
   Answer: Let x be the number that graduated. Then

$$\frac{x}{500} = 0.45$$

   So $x = (.45)(500) = 225$.

5 Percents

A **percentage** is the numerator of a ratio where the denominator is 100.
(EXAMPLE) The basketball team made 60 percent of their shots since

$$0.60 = \frac{60}{100} = 60\%$$

- To convert to a percentage, move the decimal point two places to the right.
- Don't forget the percent sign when writing a percentage. "60%" means "60 out of 100".

5. (3 pts) The unemployment rate is 8.27%. If there are 160 million Americans in the job market, how many are unemployed?
   Answer: 8.27 percent of 160 million is

$$(.0827)(160,000,000) = 13,232,000$$

6 Relative Comparison

Percentages are used to comprehend the size of something **relative** to a **reference point**. Always ask yourself what the reference point is.

(EXAMPLE) If the basketball team shot 60%, then the reference point is all the shots they attempted.

- If they attempted 30 shots, then they made 60% of 30 which is

$$(60\%)(30) = (0.60)(30) = 18$$

- If they attempted 70 shots, then they made 60% of 70 which is

$$(60\%)(70) = (0.60)(70) = 42$$

6. (4 pts) The United States has a national debt of 17 trillion dollars, and Japan has a national debt of 11 trillion dollars. The U.S. and Japan's GDP (gross domestic product) are 15 and 5.8 trillion dollars per year respectively. Compute debt to GDP ratios for the two countries. Relatively speaking, which country has a higher debt?
Answer: U.S. $\frac{17}{15} = 1.13$ and Japan $\frac{11}{5.8} = 1.90$, so Japan has a higher ratio.

7 Percent Changes

Percent changes are added or subtracted from 100% before multiplying by the reference point.

(EXAMPLE) Suppose you start with 70 marbles. You make a bet where you will either gain or lose 20% of your marbles. Twenty percent of 70 is $(.20)(70) = 14$

- An increase of 20% would give you 120% of 70 (your original 100% plus an additional 20%).

$$(100\%)(70) + (20\%)(70) = (120\%)(70) = (1.20)(70) = 84$$

- A decrease of 20% would leave you with 80% of 70 (your original 100% minus 20%).

$$(100\%)(70) - (20\%)(70) = (80\%)(70) = (0.80)(70) = 56$$

You may use simple algebra to find the original value after a percent change.

(EXAMPLE) Poindexter got a promotion that came with a 32% raise. His new salary is $ 83,160. To find out what his old salary was, solve the equation

$$(1 + .32)x = 83160$$

to get $x = \frac{83160}{1.32} = 63,000$.

7. (4 pts) A sweater is regularly $36. For a sale, the price has been marked down 25%. The new price is ☐ dollars off the original price, and the sweater's sale price is $ ☐ .
Answer: 25% of 36, is $(.25)(36) = 9$ dollars, so the sale price is $36 - 9 = 27$

8. (3 pts) Suppose a stock market index is currently 1958 points, which is a 12% drawdown from the all-time high. Find the all-time high index value.
Answer: solve $(1 - .12)x = 1958$ to get $x = 2225$

9. (3 pts) This box of Milk Duds contained 1.85 oz, which they claim was a 15 percent increase from previous packaging. How many ounces did the previous package contain?

Answer: Let x be the size of the old box, and solve "115% of x equals 1.85". We get $1.15x = 1.85$ and $x = \frac{1.85}{1.15} = 1.61$ oz. You can check that this is correct by increasing 1.61 by 15%, and you'll recover the new size of 1.85 oz.

8 More than 100%

Have you ever heard anyone say that you can't have more than 100%? This only applies if the percentage represents a proportion. Other times, depending on the nature of the reference point, percentages could exceed 100.

- The basketball team's shooting percentage is a proportion; they could not possibly make more than 100% of their attempted shots.
- Suppose a jogger set a goal of running 5 miles. Then 5 miles is the reference point. If she runs 6 miles, then the ratio

$$\frac{6}{5} = 1.20 = 120\%$$

shows that she achieved 120% of her goal. Put another way, she surpassed her goal by 20%.

- Rearranging the previous equation shows that 120% of 5 is 6.

$$(120\%)(5) = (1.20)(5) = 6$$

10. (3 pts) A social network has 6 million users, and projects a 250% increase in the next year. After a year, they expect to have ☐ million users.

    Answer: $(3.5)(6) = 21$

11. (3 pts) In 1990, the average college textbook price was \$35. In 2014, the average textbook price was \$160. During that period, textbook prices rose by ☐ dollars. Divide that answer by the 1990 price to see that prices rose by ☐ percent.

    Answer: 125 dollar increase is $125/35 = 3.57 = 357\%$

12. (4 pts) Last year, your business earned \$3000. This year, your business earned \$7500. This year's earnings equal ☐ percent of last year's, so your earnings increased by ☐ percent.

    Answer: The ratio of this year's to last year's earnings is

$$\frac{7500}{3000} = 2.5 = 250\%$$

    This is a 150% increase.

13. (3 pts) Explain why this advertisement is mathematically flawed.
    Hint: two bars in a pack is what percentage more than one bar ?

Answer: If you increase one bar by 200%, then you'd need three bars in a pack.

9 Absolute vs Relative Change

You buy a share of stock for \$24 (this is your reference point). After one year, you sell it for \$30.

- The **absolute change/difference** is the final value minus the reference point.

$$\text{final} - \text{reference} = 30 - 24 = 6$$

  So you made a 6 dollar profit.

- The **relative change/difference** is the ratio of the absolute change to the reference point.

$$\frac{\text{final} - \text{reference}}{\text{reference}} = \frac{6}{24} = 0.25$$

  So you made a 25% profit.

Let's contrast your friend's investment. He bought a different stock for \$150, and sold it for \$156.

- The absolute change is $156 - 150 = 6$, so he also made 6 dollars.
- The relative change is $\frac{156-150}{150} = 0.04$, so he made only 4% profit.

The relative change accurately reflects that your return on investment was more impressive than your friend's.

If the final value is lower than the reference point, then the changes are negative. Suppose your stock had instead dropped from \$ 24 to \$ 18. Then

- The absolute change is $18 - 24 = -6$, so you lost 6 dollars.

- The relative change is $\frac{18-24}{24} = -25\%$, so you lost 25%.

14. (4 pts) Find the percentage gain for each of these two investments.
    - Fund A: you bought it for \$75, it went up \$21, and then you sold.
      Answer: $21/75 = .28$
    - Fund B: you bought it, it went up \$17, and then you sold it for \$75.
      Answer: $17/58 = .293$
15. (4 pts) John bought a house in 2007 for \$250 thousand. Now it is appraised to be worth \$180 thousand.
    - Find the absolute change in the house's value.
      Answer: $180 - 250 = -70$ thousand dollars
    - Find the relative change in the house's value.
      Answer: $\frac{-70}{250} = -0.28 = -28\%$, so the house lost 28% of its value

10 Consecutive Changes

To apply a percent change or difference, multiply by the factor obtained by adding or subtracting from 100%.

(EXAMPLE) Suppose you buy a share of stock for \$ 24.

- The first year, it goes up 25%. The factor is $100\% + 25\% = 125\% = 1.25$. So the new value is
$$(1.25)(24) = 30$$
- The next year, it goes down 10%. The factor is $100\% - 10\% = 90\% = 0.90$. So the new value is
$$(0.90)(30) = 27$$

Note that the two years can be applied together in one calculation:

$$(0.90)(1.25)(24) = 27$$

(NOTE) When percent changes are applied successively, the results can be counter-intuitive. For example, if the the stock goes up 25% one year, and then down 25% the next year, it is natural to assume that the net change is zero. However,

$$(0.75)(1.25)(24) = 22.5$$

So the final value is lower than where it started!

16. (3 pts) Annual sales at your company have gone up 30%, up 125%, down 15% and up 40% in the last four years respectively. If the sales four years ago were \$3 million dollars, then what were the sales in the most recent year?
    Answer: Add or subtract each percentage from 100%, and then multiply by the four factors that represent the percent changes.
$$3,000,000(1.30)(2.25)(0.85)(1.40) = 10,442,250$$

17. (3 pts) Suppose an investment loses 60% of its value. Then it would need to subsequently gain [ ] percent to get back to even.

Answer: solve $(1+x)(1-.60)=1$ to get 150%

11 Reference Points

When you compare values using percentages, be sure to keep your calculations consistent with the stated reference point. In previous examples, we often had temporal changes where a quantity changed FROM one value TO another. In that case, the starting point is your reference.

(EXAMPLE) Business revenues rose from \$ 30 million to \$ 34 million. That is a relative increase of:

$$\frac{34-30}{30} = .133 = 13.3\%$$

In English, the reference point can also be indicated by what follows "of" or "than".

(EXAMPLE) A car gets 30 miles per gallon (MPG), and a truck gets 20 MPG.

- The absolute difference is 10 MPG.
- The car gets 50% more MPG **than the truck**. Since the truck is the reference point, we have

$$\frac{30-20}{20} = \frac{10}{20} = 50\%$$

- The truck gets 33% fewer MPG **than the car**. Now the car is the reference point, so

$$\frac{|20-30|}{30} = \frac{|-10|}{30} = \frac{10}{30} = 33\%$$

The negative sign is embedded in the word "fewer".

18. (3 pts) The number of newspaper subscribers decreased from 120 thousand to 75 thousand in the last decade. That is a [ ] percent decline.

Answer: $\frac{45}{120} = .375$

19. (4 pts) Peter picked 15 pecks of pickled peppers. Paul picked 11 pecks of pickled peppers.

- Peter picked [ ] percent more than Paul did.

  Answer: Paul's 11 pecks is the reference point, so Peter picked $\frac{15-11}{11} = .364 = 36.4\%$ more than Paul.

- Paul picked [ ] percent less than Peter did.

  Answer: Peter's 15 pecks is the reference point, so Paul picked $\frac{|11-15|}{15} = .267 = 26.7\%$ less than Peter.

20. (3 pts) A gallon of gasoline cost about \$3.50 in 2012. That is 185% more than it was in 1990. What was the price in 1990?
Answer: Let x be the 1990 price, which is the reference point. Then in words, the 1990 price plus 185% of the 1990 price equals today's price. This translates to the equation

$$x + 185\%x = 3.50$$

$$(100\% + 185\%)x = 3.50$$

$$2.85x = 3.50$$

Solve to get $x = \frac{3.50}{2.85} = 1.23$

12 Percentage Points

Suppose you have a \$1000 credit card balance with a 24% annual interest rate.

- To find a month's interest, divide the annual rate by 12.

$$\left(\frac{0.24}{12}\right)(1000) = 20$$

- Another bank is offering you balance transfer to a card with a 15% interest rate.
  - In absolute terms,
  $$15 - 24 = -9$$
  so you would say the new rate is 9 **percentage points** lower.
  - In relative terms,
  $$\frac{15 - 24}{24} = \frac{-9}{24} = -0.375$$
  so you would say the new rate is 37.5 **percent** lower.

21. (4 pts) Suppose the unemployment rate dropped from 7.5% to 6%.

- The absolute drop is [ ] percentage points.
  Answer: $6 - 7.5 = -1.5$, so it dropped 1.5 percentage points.
- The relative drop is [ ] percent.
  Answer: $\frac{6-7.5}{7.5} = \frac{-1.5}{7.5} = .20$, so it dropped 20 percent

22. (4 pts) A basketball player improved her shooting from 25% to 35%.

That represents a [ ] percentage point, or [ ] percent increase.
Answer: 10, 40

23. (4 pts) Suppose a new cancer treatment has been developed. The survival rate is now 35%, which is a 150% improvement over the old treatment.

(a) The old treatment's survival rate was ☐ percent.

(b) The new treatment increased survival rates by ☐ percentage points.

Answer: At first glance it seems impossible to increase by 150% and still be at 35%. But this is a percent increase, not a percentage point increase. Get the answer by solving

$$(1 + 1.5)x = 2.5x = 35$$

So $x = \frac{35}{2.5} = 14$, and the old treatment's survival rate was 14%. So the new treatment increased survival rates by 21 percentage points.