Binomial Distribution

Let your Yes be Yes, and your No be No. Anything beyond this comes from evil.
– Matthew 5:37
I know thy works, that thou art neither cold nor hot: I would thou wert cold or hot.
– Revelation 3:15

Instructions

- Read everything carefully, and follow all instructions.
- Do the numbered problems that are boxed. Show your work neatly in the allotted space.
- To receive full credit, you must justify your answer by showing your work or calculator commands.
- Circle your final answer, or write it in the spot provided.
- You may work with others, or ask for help. Your answers should reflect your own understanding of the material.
- Select, unspecified, parts of this take-home project may be graded to determine $\boxed{}$ % of your grade.
- Neatly tear the pages out of your book, and have them prepared to submit in class on the due date.
- On the due date, there will be a short in-class portion to determine $\boxed{}$ % of your grade.

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1 Bernoulli Trial

A **Bernoulli (binary) trial** is an event with two possible outcomes. The sample space Ω contains excactly two things. Common binary pairs are listed in this table:

success	1	true	yes	win	head
failure	0	false	no	lose	tail

1. (8 pts) Does the event represent a Bernoulli trial ? What are the two possible outcomes?

   - A basketball player attempts a shot.
     Answer: yes, make or miss
   - A couple finds out the gender of their baby.
     Answer: yes, male or female
   - A breeder discovers the number of puppies in a litter.
     Answer: no, more than two possiblities
   - A professor notes whether a student attended class today.
     Answer: yes, present or absent

2 Equally Likely Outcome Case: Fair Coins

Imagine flipping n coins, and let the **random variable** X be the number of heads that come up. Assume the coins are "fair", meaning that heads and tails are **equally likely**:

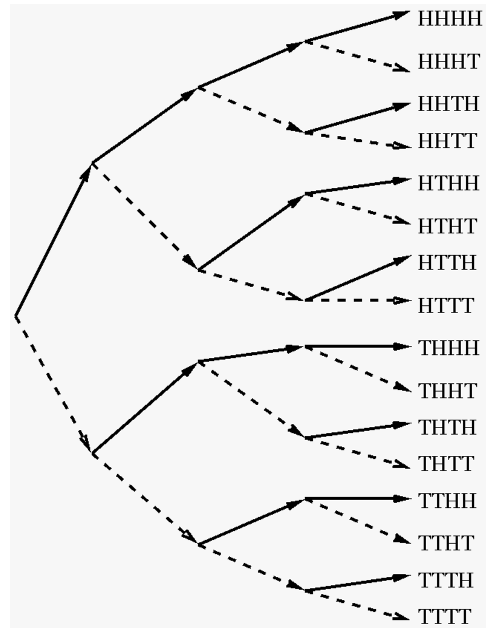
- $P(H) = 0.5$
- $P(T) = 0.5$

We can write the theoretical distribution of X as follows:

- Use a tree diagram to write down Ω.
- For each value of x (number of heads), count the ways it could happen.
- Since the outcomes are equally likely, divide by the size of Ω to get $P(x)$.

Let's do this for small values of n and look for a pattern.

2.1 $n = 1$ fair coin



There are $2^1 = 2$ outcomes in Ω:

$$\Omega = \{H, T\}$$

x	events	# ways	P(x)	CDF
0	$\{H\}$	1	$\frac{1}{2} = 0.5$	0.5
1	$\{T\}$	1	$\frac{1}{2} = 0.5$	1

2.2 $n = 2$ fair coins

There are $2^2 = 4$ outcomes in Ω:

$$\Omega = \{HH, HT, TH, TT\}$$

x	events	# ways	P(x)	CDF
0	$\{TT\}$	1	$\frac{1}{4} = 0.25$	0.25
1	$\{HT, TH\}$	2	$\frac{2}{4} = 0.50$	0.75
2	$\{HH\}$	1	$\frac{1}{4} = 0.25$	1

2.3 $n = 3$ fair coins

There are $2^3 = 8$ outcomes in Ω:

$$\Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

x	events	# ways	P(x)	CDF
0	$\{TTT\}$	1	$\frac{1}{8} = 0.125$	0.125
1	$\{HTT, THT, TTH\}$	3	$\frac{3}{8} = 0.375$	0.500
2	$\{HHT, HTH, THH\}$	3	$\frac{3}{8} = 0.375$	0.875
3	$\{HHH\}$	1	$\frac{1}{8} = 0.125$	1

2.4 $n = 4$ fair coins

There are $2^4 = 16$ outcomes in Ω:

$$\begin{aligned}\Omega \;=\; \{&HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT\\ &THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT\}\end{aligned}$$

x	events	# ways	P(x)	CDF
0	$\{TTTT\}$	1	$\frac{1}{16} = 0.0625$	0.0625
1	$\{HTTT, THTT, TTHT, TTTH\}$	4	$\frac{4}{16} = 0.2500$	0.3125
2	$\{HHTT, HTHT, HTTH, THHT, THTH, TTHH\}$	6	$\frac{6}{16} = 0.3750$	0.6875
3	$\{HHHT, HHTH, HTHH, THHH\}$	4	$\frac{4}{16} = 0.2500$	0.9375
4	$\{HHHH\}$	1	$\frac{1}{16} = 0.0625$	1

Pascal's Triangle

In the case of n coins, we see that Ω contains 2^n outcomes. Here is the pattern for number of ways each particular x could occur:

n	$x=0$	$x=1$	$x=2$	$x=3$	$x=4$	$x=5$	$x=6$	total
1	1	1						2
2	1	2	1					4
3	1	3	3	1				8
4	1	4	6	4	1			16
5	1	**5**	**10**	10	5	1		32
6	1	6	**15**	20	15	6	1	64

You might recognize this as **Pascal's triangle**. The way it is written here, each interior entry is the sum of the numbers northwest and north of it. For example, $15 = 5 + 10$.

Using this table, we do not need to list the sample space Ω to find probabilities. For example, suppose we flip $n = 4$ fair coins and want to know the probability of getting two heads. Look in row 4 to see that there are 6 ways it could happen, then divide by 2 to the 4th power.

$$P(x = 2) = \frac{6}{2^4} = \frac{6}{16} = .375$$

2. (3 pts) If you flip $n = 5$ fair coins, find $P(x = 2)$.
   Answer: $\frac{10}{32} = .3125$

3. (3 pts) If you flip $n = 6$ fair coins, find $P(x \leq 2)$.
   Answer: $\frac{22}{64} = .34375$

4. (3 pts) Write the next row ($n = 7$) of Pascal's triangle.
   Answer: 1,7,21,35,35,21,7,1

5. (3 pts) If you flip $n = 7$ fair coins, find the probability that at least 3 are heads.
   Answer: $\frac{99}{128} = .7734375$

6. (2 pts) Suppose a man tells you he has 7 grandchildren.

Assuming independence, and that boys and girls are equally likely, find the probability that he has at least 3 grandsons.
Answer: .7734375

7. (2 pts) What is the size of Ω for $n = 20$ coins?
Answer: $2^{20} = 1048576$

8. (2 pts) Suppose you apply for a job. You will either be offered the job or not. Does it therefore follow that your chances of getting the job are "50-50" ? Explain.
Answer: no, two outcomes are not necessarily equally likely

3 Not Equally Likely Outcomes Case: Basketball Shots

In the previous section, we used fair coins to study the case where both outcomes in a Bernoulli trial are equally likely. In that case,

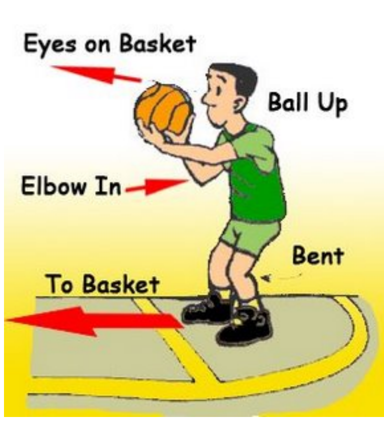
- $P(H) = 0.5$
- $P(T) = 0.5$

In this section we will generalize to the situation where one outcome may be more likely than the other. The prototypical example is shooting a basketball. Let 'Y' or 'N' indicate that the shot was made or missed respectively.

- $P(Y) = p$
- $P(N) = P(\overline{Y}) = 1 - p$

For illustration, suppose a player attempts $n = 4$ foul shots, each with $P(Y) = 0.7$ and $P(N) = 0.3$. Assume that each attempt is **independent** of the others, so that the probability of observing a sequence is the product of the probabilities for each separate shot. The random variable X will count the number of made shots. Let's list all the possible outcomes:

outcome	x	probability if $P(Y) = 0.7$	probability if $P(Y) = p$
YYYY	4	$(0.7)(0.7)(0.7)(0.7) = 0.2401$	p^4
YYYN	3	$(0.7)(0.7)(0.7)(0.3) = 0.1029$	$p^3(1-p)$
YYNY	3	$(0.7)(0.7)(0.3)(0.7) = 0.1029$	$p^3(1-p)$
YNYY	3	$(0.7)(0.3)(0.7)(0.7) = 0.1029$	$p^3(1-p)$
NYYY	3	$(0.3)(0.7)(0.7)(0.7) = 0.1029$	$p^3(1-p)$
YYNN	2	$(0.7)(0.7)(0.3)(0.3) = 0.0441$	$p^2(1-p)^2$
YNYN	2	$(0.7)(0.3)(0.7)(0.3) = 0.0441$	$p^2(1-p)^2$
YNNY	2	$(0.7)(0.3)(0.3)(0.7) = 0.0441$	$p^2(1-p)^2$
NYYN	2	$(0.3)(0.7)(0.7)(0.3) = 0.0441$	$p^2(1-p)^2$
NYNY	2	$(0.3)(0.7)(0.3)(0.7) = 0.0441$	$p^2(1-p)^2$
NNYY	2	$(0.3)(0.3)(0.7)(0.7) = 0.0441$	$p^2(1-p)^2$
YNNN	1	$(0.7)(0.3)(0.3)(0.3) = 0.0189$	$p(1-p)^3$
NYNN	1	$(0.3)(0.7)(0.3)(0.3) = 0.0189$	$p(1-p)^3$
NNYN	1	$(0.3)(0.3)(0.7)(0.3) = 0.0189$	$p(1-p)^3$
NNNY	1	$(0.3)(0.3)(0.3)(0.7) = 0.0189$	$p(1-p)^3$
NNNN	0	$(0.3)(0.3)(0.3)(0.3) = 0.0081$	$(1-p)^4$



Because multiplication is commutative, probabilities are equal whenever the number of made shots X is the same. For example, to make $x = 3$ out of four shots, we always multiply by p three times and $(1-p)$ once.

$$P(YYYN) = P(YYNY) = P(YNYY) = P(NYYY) = p^3(1-p)$$

Making 3 out of 4 shots could happen four different ways, each having identical probability. So to add up the probabilities, it is easier to simply multiply by four.

$$
\begin{aligned}
P(x=3) &= P(YYYN)+P(YYNY)+P(YNYY)+P(NYYY) \\
&= p^3(1-p)+p^3(1-p)+p^3(1-p)+p^3(1-p) \\
&= 4p^3(1-p)^1 \\
&= 4(.7)^3(.3)^1 \\
&= .4116
\end{aligned}
$$

Following this procedure, we can construct the **theoretical probability distribution** for X, using $p = 0.7$.

x	$P(x)$	CDF
0	$1(0.7)^0(0.3)^4 = 1(.0081) = .0081$	.0081
1	$4(0.7)^1(0.3)^3 = 4(.0189) = .0756$	.0837
2	$6(0.7)^2(0.3)^2 = 6(.0441) = .2646$	.3483
3	$4(0.7)^3(0.3)^1 = 4(.1029) = .4116$	.7599
4	$1(0.7)^4(0.3)^0 = 1(.2401) = .2401$	1
	1	

Recognize the pattern 1-4-6-4-1 from Pascal's triangle, and the exponents that indicate how many shots were successes or failures respectively. For a bonus point, go back to the picture that illustrates seven grandchildren, and circle your favorite.

As another example, suppose $p = .23$ and there are $n = 6$ attempts. Two successes implies four failures, and consulting the 6th row of Pascal's triangle, we can calculate:

$$P(x=2) = \boxed{15}\left(\boxed{.23}\right)^{\boxed{2}}\left(\boxed{.77}\right)^{\boxed{4}} = .279$$

9. (3 pts) Using the table above for $n = 4$ attempts with success probability 70%, how many shots do we "expect" to be made, i.e. find the theoretical mean of X. How does this answer relate to the values $n = 4$ and $p = 0.70$?
   Answer: $\mu = (0)(.0081) + (1)(.0756) + (2)(.2646) + (3)(.4116) + (4)(.2401) = 2.8 = (4)(0.7)$

10. (3 pts) Finish this list of all yes-no sequences that would result in exactly two successes in five attempts.
   $$\{YYNNN, YNYNN, YNNYN, \cdots\}$$
   How many patterns are in the list?
   Answer: 10

11. (3 pts) A basketball player attempts $n = 5$ three point shots, each with probability $p = 0.4$ of success. Fill in the blanks to find the probability that she makes exactly two of the five shots.
   $$\boxed{}\left(\boxed{}\right)^{\boxed{}}\left(\boxed{}\right)^{\boxed{}} =$$
   Answer: $10(0.4)^2(0.6)^3 = 0.3456$

12. (3 pts) If you flip seven fair coins, find the probability that you get exactly 3 heads.

$$\boxed{}\left(\boxed{}\right)^{\boxed{}}\left(\boxed{}\right)^{\boxed{}} =$$

Answer: $35(.5)^3(.5)^4 = .273$

13. (3 pts) If you roll seven fair dice, find the probability that you get exactly 3 "sixes". Hint: what is the value of p?

$$\boxed{}\left(\boxed{}\right)^{\boxed{}}\left(\boxed{}\right)^{\boxed{}} =$$

Answer: $p = 1/6$, so $P(x = 3) = 35(1/6)^3(5/6)^4 = .0781$

14. (3 pts) If you apply for eight jobs, and you have a 12% chance of being offered each particular job. Find the probability that you get exactly one job offer.

$$\boxed{}\left(\boxed{}\right)^{\boxed{}}\left(\boxed{}\right)^{\boxed{}} =$$

Answer: $8(.12)^1(.88)^7 = .392$

4 Binomial Distribution

Consider a sequence of n Bernoulli trials, each independent of the others, with probability p of success. The **binomial** random variable X counts the number of successes, and we say it has the **binomial distribution** with parameters n and p, which is denoted by:

$$X \sim BI(n,p)$$

In previous sections we have examined cases such as:

- flipping $n = 3$ coins with $p = 0.5$, so the number of heads has the distribution $X \sim BI(3, 0.5)$
- shooting $n = 4$ shots with $p = 0.7$, so the number of made shots has the distribution $X \sim BI(4, 0.7)$

There are four key features that should help you recognize when a binomial **model** is appropriate:

- Bernoulli (binary) trials, each with two outcomes.
- Sequence of n trials (or the experiment will be repeated n times).
- Outcomes of the trials are **independent** of each other.
- Each trial is **identical**, having the same probability p of success.

Theoretical Statistics

Mathematicians have used the binomial distribution's theoretical probabilities to derive formulas for

- **mean (expected value):** $\mu = np$
- **standard deviation:** $\sigma = \sqrt{np(1-p)}$

For our basketball shooting example $BI(4, 0.7)$, we get $\mu = (4)(0.7) = 2.8$ and $\sigma = \sqrt{(4)(.7)(.3)} = 0.917$. These values for μ and σ can also be obtained from your calculator using *1-varstats L1,L2*.

15. (2 pts) Is the binomial distribution continuous or discrete?
Answer: discrete

16. (2 pts) If $n = 100$ and $p = 0.2$, find the mean and standard deviation.
Answer: $\mu = 100(.2) = 20$, $\sigma = \sqrt{100(.2)(.8)} = \sqrt{16} = 4$

17. (2 pts) If $n = 12$ and $p = 0.9$, find the mean and standard deviation.
Answer: $\mu = 12(.9) = 10.8$, $\sigma = \sqrt{12(.9)(.1)} = 1.039$

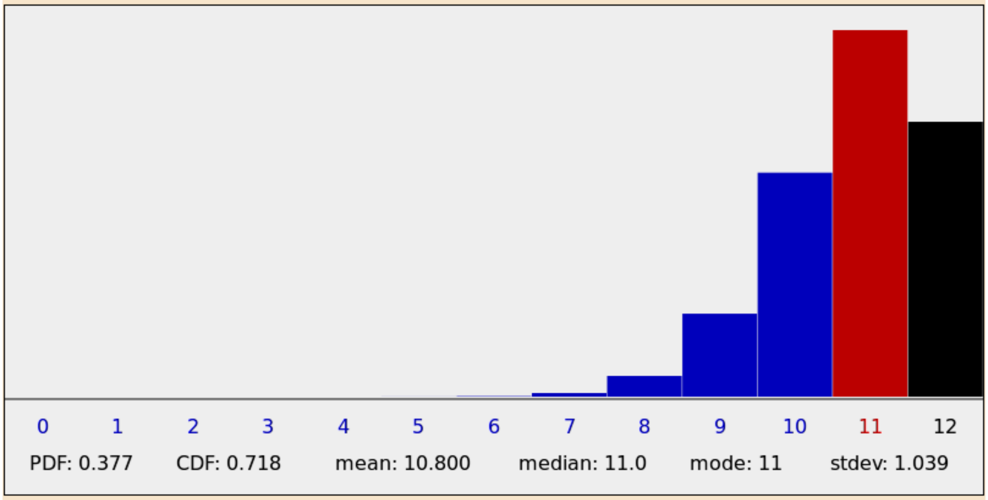
18. (2 pts) If $n = 24$ and $p = \frac{1}{6}$, find the mean and variance.
Answer: $\mu = 24(1/6) = 4$, $\sigma^2 = 24(1/6)(5/6) = 3.33$

19. (2 pts) Which has a higher variance, $BI(82, 0.5)$ or $BI(82, 0.75)$?
Answer: when $\sigma = .5$

Binomial Distribution Table

Computers can produce PDF and CDF values using the logic descibed for the basketball shooting example. Here is a screenshot from the web page `http://massey.limfinity.com/apps/binomial.php`, which has calculated probabilities and statistics for $X \sim BI(12, 0.9)$.

Let X ~ BI(n,p) **n:** 12 **p:** 0.9 **x:** 11



x	P(x)	CDF
0	0.0000	0.0000
1	0.0000	0.0000
2	0.0000	0.0000
3	0.0000	0.0000
4	0.0000	0.0000
5	0.0000	0.0001
6	0.0005	0.0005
7	0.0038	0.0043
8	0.0213	0.0256
9	0.0852	0.1109
10	0.2301	0.3410
11	0.3766	0.7176
12	0.2824	1.0000

Here is some information that can be determined from the screenshot.

- $P(x = 11) = PDF(11) = 0.3766$
- $P(x \leq 11) = CDF(11) = 0.7176$
- $P(x > 9) = 1 - CDF(9) = 1 - 0.1109 = 0.8891$
- The mean is 10.8.
- The variance is $\sigma^2 = (1.039)^2 = 1.08$.
- The five number summary is $(0, 10, 11, 12, 12)$.
- The mode is $x = 11$.
- The distribution is left skewed.

T.I. Calculator

You can use calculator functions to find specific entries from a binomial distribution table.

- $\boxed{\text{2nd}}$ $\boxed{\text{DISTR}}$, scroll down to find "binompdf"
- $\boxed{\text{2nd}}$ $\boxed{\text{DISTR}}$, scroll down to find "binomcdf"

Let's illustrate with $X \sim BI(12, 0.9)$. It may help to write out possible values of X, and box in the cases you want included in the answer. Compare the calculator's answers to those obtained from the previous table.

- probability X is "exactly" 10

  $0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ \boxed{10}\ 11\ 12$

  $P(X = 10) = \mathit{binompdf}(12, .9, 10) = .2301$
- probability X is 10 "or fewer"

  $\boxed{0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10}\ 11\ 12$

  $P(X \leq 10) = \mathit{binomcdf}(12, .9, 10) = .3410$
- probability X is "at least" 10

  $0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ \boxed{10\ 11\ 12}$

  $P(X \geq 10) = 1 - \mathit{binomcdf}(12, .9, 9) = .8891$
- probability X is "between" 8 and 10

  $0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ \boxed{8\ 9\ 10}\ 11\ 12$

  $P(8 \leq X \leq 10) = \mathit{binomcdf}(12, .9, 10) - \mathit{binomcdf}(12, .9, 7) = .3367$

20. (10 pts) A basketball player takes 20 foul shots, each with a 75% chance of success.

    (a) Let X be the number of made shots. What distribution models X ?
    Answer: $X \sim BI(20, 0.75)$

    (b) Find the probability that she makes exactly 75% of her shots.
    Answer: $\mathit{bipdf}(20, .75, 15) = .202$

    (c) Find the probability that she makes 75% or fewer of her shots.
    Answer: $\mathit{bicdf}(20, .75, 15) = .585$

    (d) Find the probability that she makes at least 75% of her shots.
    Answer: $1 - \mathit{bicdf}(20, .75, 14) = .617$

21. (20 pts) Create a table on `http://massey.limfinity.com/apps/binomial.php`, and use it to answer these questions about $X \sim BI(70, 0.4)$. When possible, check the answers with your calculator.

    (a) The expected value of X is $\boxed{\quad}$ and the variance is $\boxed{\quad}$.

    Answer: $\mu = 28$ and $\sigma^2 = 16.8$

    (b) The median is $\boxed{\quad}$, and the inter-quartile range is $\boxed{\quad}$.

    Answer: 28, $31 - 25 = 6$

(c) $x = 36$ is the $\boxed{}$ th percentile.

Answer: 98th

(d) The 36th percentile is $\boxed{}$.

Answer: $x = 26$

(e) The probability that X is at exactly 30, i.e. $P(X = 30)$.
Answer: $bipdf(70, .4, 30) = .0853$

(f) The probability that X is at most 30, i.e. $P(X \leq 30)$.
Answer: $bicdf(70, .4, 30) = .7306$

(g) The probability that X is more than 30, i.e. $P(X > 30)$.
Answer: $1 - bicdf(70, .4, 30) = .2694$

(h) The probability X is between 25 and 35, i.e. $P(25 \leq X \leq 35)$
Answer: $bicdf(70, .4, 35) - bicdf(70, .4, 24) = .768$

5 Applications

Example: Car Salesman

Suppose a car salesman accosts 40 potential customers per week, and each one has (independently) a 9% chance of buying a car.
Model the number of cars sold as $X \sim BI(40, 0.09)$.

Here is a simulation of 200 obervations of this random variable, showing how many cars he sold each week. Notice that he will have good weeks and bad weeks, attributable entirely to chance. For a bonus point, circle the two best weeks and two worst weeks in this simulated list of 200 weekly car sales:

```
> rbinom(200,40,.09)

 6  4  3  4  1  3  2  5  0  7  2  2  4  2  2  6  5  4  5  6  2  2  2  6  4
 3  1  4  2  4  3  8  3  4  2  5  4  6  3  4  6  3  1  2  7  2  5  5  7  2
 6  3  3  2  7  3  3  5  4  6  4  3 10  5  1  3  5  5  7  4  1  4  5  2  3
 4  2  2  2  1  5  3  5  4  6  3  4  2  4  4  2  3  2  4  3  3  1  3  2  5
 2  2  7  4  3  5  1  5  3  4  7  7  4  4  2  4  1  8  5  3  2  2  3  2  5
 6  4  3  3  5  1  6  3  2  2  4  3  7  4  3  2  5  5  4  2  2  5  2  3  5
 4  1  6  0  1  7  5  4  8  4  4  2  3  5  1  3  1  3  3  3  9  4  5  5  2
 4  5  2  2  1  5  2  3  2  3  2  3  2  1  1  2  3  2  4  5  2  3  5  7  3
```

Theoretically, we know that:

- He "expects" to sell an average of $\mu = (40)(.09) = 3.6$ cars per week.
- The standard deviation is $\sigma = \sqrt{(40)(.09)(.91)} = 1.81$ cars.
- The probability that he sells fewer than 3 cars is

$$\mathit{binomcdf}(40, .09, 2) = .2894$$

- The probability that he sells more than 3 cars is

$$1 - \mathit{binomcdf}(40, .09, 3) = .4908$$

- The probability that he sells zero cars is

$$\mathit{binompdf}(40, .09, 0) = .0230$$

- The probability that he sells between 2 and 5 cars is

$$\mathit{binomcdf}(40, .09, 5) - \mathit{binomcdf}(40, .09, 1) = .7395$$

Example: Mortgage Defaults

Suppose a mortgage backed security is based on 1000 similarly valued subprime home mortgages, each having a 1 in 40 chance of default. A binomial model for the number of defaults is

$$D \sim BI(1000, 0.025)$$

An investment bank may use this model to calculate things such as:

- The expected number of defaults is $\mu = (1000)(.025) = 25$.
- The probability that the number of defaults is in the 20's is

$$P(20 \leq D \leq 29) = \mathit{binomcdf}(1000, .025, 29) - \mathit{binomcdf}(1000, .025, 19) = .690$$

- Perhaps the security loses money if more than 4% of the loans default. The probability of this is

$$P(D > 40) = 1 - \mathit{binomcdf}(1000, .025, 40) = .00178$$

A bank manager that uses this model may be convinced that this security is a great investment. With such a small chance of losing money, the bank should do as many of these deals as possible! In fact, this is what many banks, pension funds, insurance companies, and other organizations thought prior to the 2008 financial crisis. They were wrong, lost a lot of money, and often required government bailouts. After the housing bubble burst, the economy tanked and many people could not or would not repay their home loans. Defaults were revealed to be **non-independent** events, due to the negative feedback loop inherent in modern economies.

Distributions that assume independence, such as the binomial described above, were very bad models for investment risks. The bubble, and the subsequent unemployment and recession can be partially blamed on the blind use of inappropriate probability models. You can do all the math correctly, but if you are using the wrong model, the results can be very misleading. The binomial distribution should not be used when the trial results are not independent, as is the case whenever you are studying **contagious** phenomena. For a bonus point, put a giant X through the binomial model and calculations above.

22. (4 pts) A website displays ads to generate revenue. Suppose the ad click-through rate (CTR) is 0.48 %, meaning that 0.48% of displayed ads get clicked. Furthermore, each ad click generates \$ 0.55 in revenue. If an ad is displayed one million times, the expected number of clicks is $\boxed{\quad\quad}$, which corresponds to a revenue of $\boxed{\quad\quad}$ dollars.

    Answer: $(.0048)(10000000) = 4800$, which is $(4800)(.55) = 2640$ dollars

23. (3 pts) You are hosting a dinner for 35 guests. Some (you estimate about 60%) will want something cold to drink; others will prefer something hot. There are only 24 cold ones in the refrigerator. Model the number of cold drinks requested by $X \sim BI(35, 0.6)$. Find the probability that you will have enough cold ones to meet demand.
    Answer: $P(x \leq 24) = \mathit{bicdf}(35, .6, 24) = .888$

24. (12 pts) A basketball group has 18 players on the roster. Due to scheduling conflicts, each individual misses a third of the time (assume independence). Model the number of players that show up to play as a binomial random variable and answer these questions:

(a) State the value of p, the probability that an individual player **shows up** to play.
Answer: $p = 2/3$

(b) What is the "expected" turnout?
Answer: $(18)(2/3) = 12$

(c) Find the probability that exactly ten players show up.
Answer: $\mathit{binompdf}(18, 2/3, 10) = .116$

(d) Find the probability that fewer than ten show up.
Answer: $\mathit{bicdf}(18, 2/3, 9) = .108$

(e) Find the probability that at least ten show up.
Answer: $1 - \mathit{bicdf}(18, 2/3, 9) = .892$

(f) Find the probability that between ten and fifteen show up.
Answer: $\mathit{bicdf}(18, 2/3, 15) - \mathit{bicdf}(18, 2/3, 9) = .860$

25. (6 pts) In the U.S, about 51.25% of newborns are male.

(a) If a hospital delivers 100 babies, find the probability that a majority are boys.
Answer: $P(x > 50) = 1 - \mathit{bicdf}(100, .5125, 50) = .560$

(b) If a hospital delivers 1000 babies, find the probability that a majority are boys.
Answer: $P(x > 500) = 1 - \mathit{bicdf}(1000, .5125, 500) = .776$

(c) If a hospital delivers 10000 babies, find the probability that a majority are boys.
Answer: $P(x > 5000) = 1 - \mathit{bicdf}(10000, .5125, 5000) = .994$

26. (10 pts) For each situation, are the trials reasonably independent events, so that the binomial is a good model? Explain your reasoning.

(a) An official wants to model the number of children that fall during a race.
Answer: not independent, cascades

(b) A hospital wants to model the number of liver transplant patients to survive at least one year.
Answer: reasonably independent

(c) A college dorm manager wants to model the number of rooms that get b
Answer: not independent, contagious

(d) A foreman wants to model the number of isolated machines that will break during a shift.
Answer: reasonably independent

(e) A college administrator wants to model the retention of freshmen students (how many will return to the school as sophomores).
Answer: not independent, peers

27. (12 pts) A cell phone distributor sells 300 insurance policies to owners of a certain smartphone. Each policy costs \$100. If the phone is lost or destroyed, the company will replace it at a cost of \$500. Here is a table describing the phone company's point of view:

result	revenue	cost	net	probability
must replace phone	100	500	-400	0.15
no claim made	100	0	100	0.85

(a) Find expected profit on one insurance policy.
Answer: $(-400)(.15) + (100)(.85) = 25$

(b) Find expected profit on 300 such policies.
Answer: 7500

(c) How many phones should they "expect" to replace?
Answer: $(.15)(300) = 45$

(d) The company will make a profit selling these insurance policies as long as fewer than $\boxed{}$ out of 300 phones must be replaced. Hint: let X be the number of phones that must be replaced, and set revenue equal to cost to find the break-even value of X.
Answer: solve $500x = (100)(300)$ to get break-even of $\frac{(300)(100)}{500} = 60$ phones, so profit if fewer than 60

(e) Model X by a binomial distribution to find the probability that the company makes a profit on these 300 policies.
Answer: $\mathit{bicdf}(300, .15, 59) = .988$

28. (5 pts) To build a robot requires 6 sprockets and 9 cogs.
You have 6 sprockets and 11 cogs in stock.
From experience you know that 4% of sprockets and 10% of cogs are faulty. Find the probability that a robot can be built from current inventory.

Answer: must have zero faulty sprockets, and 2 or fewer faulty cogs
$\mathit{bipdf}(6, .04, 0) * \mathit{bicdf}(11, .1, 2) = .713$