## Carson-Newman University MATH 201 Test 1, Spring 2020

Directions:

- You may use a stand-alone calculator, but electronic communication is prohibited. No other sources of information are allowed.

- To receive full credit, you must show all relevant work to justify your answer on the test paper. Write down your calculator commands for statistical calculations.

- Clearly identify your final answer, correct to at least 3 significant digits.

- Use notation as described in class.

Honor Pledge: I pledge that I will neither give nor receive unauthorized help on this test from any person, technology, or other resource, and that I will abide by the honor code of Carson-Newman University.

Signed: _____
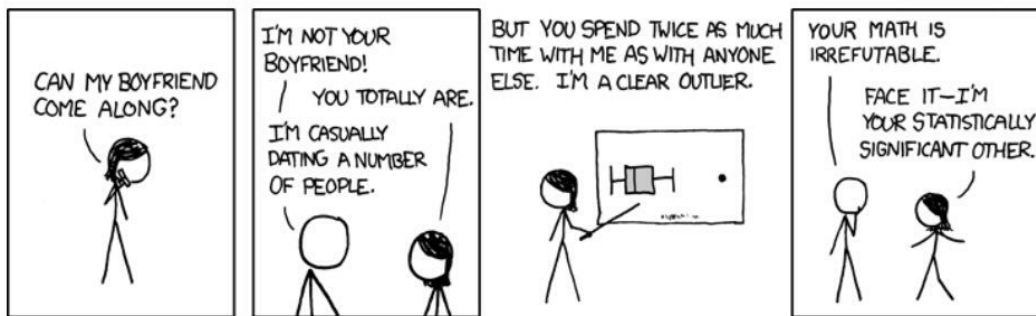
1. The New England Confectionery will sell 8.65 billion "Conversation" hearts this year in the U.S. That is approximately [ ] hearts per-capita.

    Answer: $\frac{8,650,000,000}{330,000,000} = 26.2$ hearts per person

2. Label each random variable as either discrete or continuous.

    (a) The number of Valentine's Day cards you receive.
    Answer: discrete

    (b) The calories consumed at a Valentine's dinner.
    Answer: continuous

    (c) The duration of a hug or kiss.
    Answer: continuous

3. A restaurant chain recorded the time spent by couples at Valentine's dinner: that is from the time they were seated until the time they left the table.

   Let the random variable x be the number of minutes at the table. Use this Excel screenshot to answer the questions about the empirical distribution of x.

   (a) sample size
       Answer: $n = 2100$

   (b) mean
       Answer: $\bar{x} = \frac{164471}{2100} = 78.3$

   (c) median
       Answer: 70

   (d) mode
       Answer: 59

   (e) left skewed or right skewed ?
       Answer: right-skew

   (f) 5-number summary
       Answer: 43,59,70,97,158

   (g) range
       Answer: $158 - 43 = 115$

   (h) inter-quartile range
       Answer: $97 - 59 = 38$

   (i) find the 60'th percentile
       Answer: 80, since $CDF(80) = .60381$

   (j) $x = 60$ minutes is the _____ percentile
       Answer: 29th

   (k) what proportion spent at least 90 minutes ?
       Answer: $1 - CDF(89) = .316$

   (l) what proportion spent 60-89 minutes ?
       Answer: $CDF(89) - CDF(59) = .684 - .271 = .413$

   (m) If the variance is 576, find the z-score of $x = 60$.
       Answer: $z = \frac{60 - 78.3}{\sqrt{576}} = -.7625$

4. At a Valentine's dance, the DJ has queued up $n = 5$ love songs that have the following play times:

$$\{ \; 3:08, \; 4:41, \; 3:52, \; 2:47, \; 4:12 \; \}$$

   (a) The mean run time is $\bar{x} = $ [   ] minutes and [   ] seconds.

       Answer: 226 seconds $= 3:44$

   (b) The standard deviation is [   ] seconds.

       Answer: 1varstats gives $s = 46.5$ seconds

5. Here are the Valentine's Day high temperatures in Knoxville from 1950 through 2019.

stem	leaves
2	78
3	04566778
4	1112222334556678
5	00001122333444555555577
6	0111222344667889
7	1123

(a) State the sample size n.

Answer: $n = 70$

(b) If $\sum x = 3628$, find the mean temperature.

Answer: $\bar{x} = \frac{3628}{70} = 51.8$ degrees

(c) Find the median temperature.

Answer: the 35th and 36th entries are both 53 degrees.

(d) Find the mode temperature.

Answer: 55 occurs 8 times

(e) You want to take your special someone on a romantic picnic, but only if it is at least $60°$. Based on the historical data, the Feb. 14 high temperature is at least $60°$ in ⬚ % of years.

Answer: count on stem plot 20 times it was at least 60, so $20/70 = .286 = 28.6\%$

6. Every year, Antonio buys his wife Sheva a dozen roses for Valentine's day. Five years ago, he could get them for $ 48. Now they cost $ 70. Compute the average annually compounded percentage increase in the price of roses.

Answer: $(70/48)^{1/5} = 1.0784$ so 7.84% annually

7. A survey asked college seniors how many people they had romantically told "I love you."

X	frequency	relative freq	cumulative freq
0	96		.219
1		.347	
2	84		
3		.107	.865
4		.064	.929
⋮	⋮	⋮	⋮

(a) The sample size is ☐ (round n to the nearest integer). Show how you found it.

Answer: $96 = .219n$ so $n = 96/.219 = 438$

(b) Fill out the missing entries in the table up to row $X = 4$.

Answer:

X	frequency	relative freq	cumulative freq
0	96	.219	.219
1	152	.347	.566
2	84	.192	.758
3	47	.107	.865
4	28	.064	.929
⋮	⋮	⋮	⋮

(c) ☐ percent of respondents had said "I love you" to at least 4 people.

Answer: $1 - cdf(3) = .135 = 13.5\%$

(d) (Bonus) Is it true that $\bar{x} > 1.5$? Explain how you know for sure, given the data above.
Answer: even if the remaining $438 - 407 = 31$ people had been $X = 5$, the mean would be $\bar{x} = 738/438 = 1.66$

8. This table shows the distribution of individuals' cupcake consumption at a Valentine's party.

cupcakes	frequency
0	
1	12
2	
3	8
	40

(a) The mean is $\bar{x} = 1.75$ cupcakes per person. How many people ate exactly 2 cupcakes?
   Answer: solve $1.75 = \frac{12+2x+24}{40}$ to get $x = 17$

(b) Find the standard deviation of the number of cupcakes eaten. If you don't know the answer to part (a), use 15.
   Answer: with $x = 17$ 1varstats L1,L2 gives $s = .870$; with $x = 15$ it gives $s = .949$

9. Matchmakers have two different scales for measuring the potential for romance between two individuals:

   - Sparks Index (SI): mean 500, standard deviation 60
   - Chemistry Quotient (CQ): mean 100, standard deviation 24

   Both SI and CQ have symmetric distributions with similar shape.

(a) Poindexter and Bertha have a Sparks Index of 635. Find the z-score.
   Answer: $z = \frac{635-500}{60} = 2.25$

(b) Translate their Sparks Index score to a corresponding Chemistry Quotient score.
   Answer: solve $2.25 = \frac{x-100}{24}$ to get $x = 154$

10. For Valentine's Day, T.I. gave his girl a shopping spree, as in his hit single "Whatever You Like":
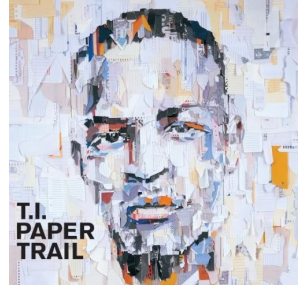
♫ My chick can have what she want.
And go in every store for any bag she want. ♫

She made purchases from six stores. Make up dollar amounts that satisfy the following conditions:

- mean $900 per store
- median $200
- minimum $100
- range $2500
- no two values are the same

Put your values in sorted order in these boxes.

Answer: many answers possible, e.g. 100,150,175,225,2150,2600

11. For Valentine's Day, T.I. gives his girlfriend a shopping spree, as in his hit single "Whatever You Like":

♫ My chick can have what she want.
And go in every store for any bag she want. ♫

On second thought, T.I. decides to give his chick some conditions:

- She can shop in $n = 6$ stores.
- The "average" money spent in the stores has to be under $200.

T.I. wasn't specific about which "average" he will compute.

If you were T.I.'s chick, would you rather the mean or the median have to stay under $200 ?

Write a paragraph explaining your answer, using an example to illustrate.

Answer: She would rather keep the median under 200 dollars. Then she could spend unlimited amount in two stores, as long as the median stays below 200, e.g. $199, 199, 199, 1000, 90000$. If the mean had to stay under $200, then she could spend no more than a total budget of $1200 in the 5 stores.

It is possible that she would prefer the mean stay under $200, but only if there were three stores in which she wanted to spend more than $200, e.g. 20,50,250,300,350. But she would be sacrificing the really high dollar purchases to do that.