

## Test 1 Practice

1. Do ALL your homework problems, rework your quizzes, and review problems done in class.
2. Suppose that high temperatures for a particular day and city have been recorded for 200 years. If the distribution is bell shaped, with  $\bar{x} = 75$ , and five number summary (45, 69, 76, 81, 96), then
  - (a) find the mean  
**Answer:** 75
  - (b) find the median  
**Answer:** 76
  - (c) find the inter-quartile range  
**Answer:**  $81 - 69 = 12$
  - (d) if a 90 degree temperature has a  $z$ -score of 1.67, then find the variance  
**Answer:** solve  $1.67 = \frac{90-75}{s}$  to get  $s = 9$ , so the variance is  $s^2 = 81$
  - (e) Sketch the boxplot that goes with the five number summary.
3. Consider the annual income distribution 5 number summaries for these two nations. (units are in thousands of dollars)
  - Akhbaristan (0, 20, 35, 50, 300)
  - Bahavia (0, 10, 20, 30, 2000)

Suppose that the mean income is  $\bar{x} = 40$  for both countries.

- (a) Which country probably has a higher standard deviation?  
**Answer:** Bahavia - more right-skewed, super rich people
  - (b) Which country probably has more people living in poverty?  
**Answer:** Bahavia - the bottom quartile is below \$10 thousand
  - (c) Which country has a bigger middle class (incomes between 30 and 60 thousand) ?  
**Answer:** In Akhbaristan, the entire 3rd quartile is middle class, plus some of the 2nd and 4th quartiles. In Bahavia, only part of the 4th quartile is middle class.
4. Here is a selection of computer rankings of the UT Mens' basketball team.

{39, 34, 42, 38, 31, 41, 21, 52, 34, 37, 26, 31, 19, 33, 33, 27}

Use your calculator to find:

- (a) Is this random variable discrete or continuous?  
**Answer:** discrete - a ranking could be 1st, 2nd, 3rd, etc, but not e.g. 3.71394
  - (b) Find the mean.  
**Answer:** 33.6
  - (c) Find the median.  
**Answer:** 33.5
  - (d) Find the standard deviation.  
**Answer:** 8.21
5. Use the attached Excel worksheet to answer these questions about the distribution of finishing position for Tony Stewart's career in NASCAR Cup races.
    - (a) Find the mean.  
**Answer:**  $5938/474 = 12.5$

- (b) Find the median.  
**Answer:** 8
- (c) Find the mode.  
**Answer:** 1
- (d) Write the 5 number summary. And sketch a boxplot  
**Answer:** 1,4,8,19,43
- (e) Find the inter-quartile range.  
**Answer:**  $19-4=15$
- (f) How would you describe the shape of the distribution?  
**Answer:** right-skewed
- (g) The variance is 125. Find the standard deviation.  
**Answer:**  $\sqrt{125} = 11.2$
- (h) What percentage of the races did he finish 2nd ?  
**Answer:**  $PDF(2) = 8.44\%$
- (i) What percentage of the races did he finish better than 25th ?  
**Answer:**  $CDF(24) = 82.91\%$
- (j) What percentage of the races did he finish worse than 10th ?  
**Answer:**  $1 - CDF(10) = 1 - .5717 = 42.83\%$

A	B	C	D	E
Position	Frequency	Position*Frequency	PDF	CDF
1	47	47	0.0992	0.0992
2	40	80	0.0844	0.1835
3	26	78	0.0549	0.2384
4	25	100	0.0527	0.2911
5	28	140	0.0591	0.3502
6	24	144	0.0506	0.4008
7	30	210	0.0633	0.4641
8	21	168	0.0443	0.5084
9	20	180	0.0422	0.5506
10	10	100	0.0211	0.5717
11	11	121	0.0232	0.5949
12	14	168	0.0295	0.6245
13	9	117	0.0190	0.6435
14	10	140	0.0211	0.6646
15	12	180	0.0253	0.6899
16	7	112	0.0148	0.7046
17	13	221	0.0274	0.7321
18	6	108	0.0127	0.7447
19	6	114	0.0127	0.7574
20	4	80	0.0084	0.7658
21	6	126	0.0127	0.7785
22	9	198	0.0190	0.7975
23	8	184	0.0169	0.8143
24	7	168	0.0148	0.8291
25	9	225	0.0190	0.8481
26	9	234	0.0190	0.8671
27	7	189	0.0148	0.8819
28	4	112	0.0084	0.8903
29	4	116	0.0084	0.8987
30	3	90	0.0063	0.9051
31	2	62	0.0042	0.9093
32	2	64	0.0042	0.9135
33	4	132	0.0084	0.9219
34	4	136	0.0084	0.9304
35	4	140	0.0084	0.9388
36	3	108	0.0063	0.9451
37	2	74	0.0042	0.9494
38	3	114	0.0063	0.9557
39	5	195	0.0105	0.9662
40	3	120	0.0063	0.9726
41	7	287	0.0148	0.9873
42	2	84	0.0042	0.9916
43	4	172	0.0084	1.0000
totals	474	5938	1.0000	

6. Work backwards from the CDF column to fill out this distribution:

$X$	freq	relative	cumulative
6			.048
7			.148
8			.376
9			.700
10			1
	250		

**Answer:**

$X$	freq	relative	cumulative
6	12	.048	.048
7	25	.100	.148
8	57	.228	.376
9	81	.324	.700
10	75	.300	1
	250	1	

(a) mean

**Answer:**  $\bar{x} = 8.728$

(b) median

**Answer:** 9

(c) mode

**Answer:** 9

(d) shape

**Answer:** left skew

7. A bored police officer accumulated these frequencies for driver speeds on I-40.

speed	freq	relative freq	cumulative freq
55-59	4		
60-64	11		.110
65-69			
70-74	43		
75-79	27		
80-84	13		

(a) Fill in the rest of the table.

**Answer:** solve  $15/n = .110$  to get  $n = 136$ , then

speed	freq	relative freq	cumulative freq
55-59	4	.029	.029
60-64	11	.081	.110
65-69	38	.279	.389
70-74	43	.316	.706
75-79	27	.199	.904
80-84	13	.096	1
	136		

(b) Using bin midpoints, find the mean and standard deviation of driver speeds.

**Answer:**  $\bar{x} = 71.3$  and  $s = 5.999$

(c) A car's  $z$ -score is  $z = -1.5$ . What is its speed?

**Answer:** solve  $-1.5 = \frac{x-71.3}{6}$  to get  $x = 62.3$

(d) What proportion of the cars are going at least 70 MPH?

**Answer:**  $\frac{83}{136} = .61$

8. Consider this stem plot for a random variable  $x$ .

1		8
2		156
3		035779
4		13368

(a) Describe the distribution's shape.

**Answer:** left-skewed

(b) Find the mean.

**Answer:**  $(18 + 21 + 25 + 26 + 30 + 33 + 35 + 37 + 37 + 39 + 41 + 43 + 43 + 46 + 48)/15 = 34.8$

(c) Find the median without using your calculator.

**Answer:** the 8th value is 37

(d) What proportion are under 30?

**Answer:**  $4/15 = .267$

9. This frequency table lists the analyst ratings for Apple stock.

stars	frequency
*	1
**	1
***	4
****	
*****	14
total	

(a) If the relative frequency for 5 stars is .359, then how many analysts were there, and how many rated the stock 4 stars?

**Answer:** solve  $\frac{14}{n} = .359$  to get  $n = 39$ , so 19 gave it 4 stars

(b) What is the mode of this distribution?

**Answer:** 4 stars

(c) \_\_\_\_\_ percent more analysts gave four stars than gave it five stars.

**Answer:**  $\frac{19-14}{14} = .357 = 35.7\%$

(d) Calculate  $\bar{x}$  by hand.

**Answer:**  $\bar{x} = \frac{(1)(1)+(1)(2)+(4)(3)+(19)(4)+(14)(5)}{39} = \frac{161}{39} = 4.128$

(e) Use your calculator to find  $\bar{x}$  and  $s$ .

**Answer:**  $\bar{x} = 4.128$ ,  $s = .894$

(f) Sketch a histogram and describe the shape of the distribution.

**Answer:** left-skewed, even though mean  $>$  median

10. For an eight hour shift, which worker, a cook or a waitress, would have a higher standard deviation in total income? Explain your answer using a complete sentence.

**Answer:** The waitress has a higher stdev since she rels on tips, and her pay is more variable/less consistent. The cook's st dev is probably zero since he makes a fixed wage.

11. In a certain class, the final grade is the weighted average of three components. Here are the weights, and the grades for a particular student.

component	weight	grade
term papers	45%	92
tests	35%	70
final exam	20%	?

What grade must the student get on her final exam to achieve a “B” in the class (minimum average of 79.5) ?

**Answer:** solve  $.45(92) + .35(70) + .20x = 79.5$  to get  $x = 68$

12. Here is a frequency table for Super Bowl MVP’s

Position	frequency
Quarterback	22
Runningback	7
Wide Receiver	6
Defensive end	2
Linebacker	2
Safety	2

- (a) Is random variable quantitative or qualitative?

**Answer:** qualitative

- (b) Sketch a barchart for this data.

- (c) Do you think it makes sense to say the distribution is right-skewed?

**Answer:** no, because qualitative data could be ordered in any way, so discussing shape makes no sense.

13. A survey of hospitals gives the frequency distribution for the age of a certain piece of medical equipment:

age	frequency
0	50
1	48
2	45
3	38
4	25
5	13
6	5
7	1

- (a) What is the mean age?

**Answer:** 2.02

- (b) What is the standard deviation of the age?

**Answer:** 1.65

- (c) What percentage of the devices are at least 4 years old?

**Answer:**  $44/225 = .196$

- (d) Sketch a histogram for this distribution, then describe the shape.

**Answer:** right skewed

14. Suppose the distribution of rocket scientist IQ’s has mean 120 and standard deviation 9. For pro athletes the mean is 95 and the standard deviation is 17. Compute  $z$ -scores for

- a rocket scientist with an IQ of 136

**Answer:**  $z = (136 - 120)/9 = 1.78$

- an athlete with an IQ of 126  
**Answer:**  $z = (126 - 95)/17 = 1.82$

Relative to the others in their respective professions, which has a more unusually high IQ?

**Answer:** the athlete since his  $z$ -score is higher

15. Suppose a team's running backs were to run for the yards indicated in this stem plot:

0	0001222333444555566779
1	0138
2	17
3	4

- (a) Find the total yards,  $\sum x_i$ .

**Answer:** 222

- (b) Find the mean.

**Answer:**  $222/30 = 7.4$

- (c) Find the median.

**Answer:** 5

- (d) Find the inter-quartile range.

**Answer:**  $9 - 3 = 6$

- (e) Describe the shape of this distribution.

**Answer:** right-skewed

- (f) What is the empirical proportion of runs that are more than 5 yards?

**Answer:**  $12/30 = .4$

16. Among Cal Tech undergraduates, the male:female ratio is 1.58 to 1. If there are a total of 965 undergraduate students, how many of them are female?

**Answer:** The female proportion is  $\frac{1}{1+1.58} = .3876$ . So there are  $(.3876)(965) = 374$  females

17. At a football game, 70 thousand fans were in attendance. The concession stands reported a revenue of \$393,000. That amounts to \_\_\_\_\_ dollars per capita.

**Answer:**  $393000/70000 = 5.61$

18. Between 1979 and 2010, Virginia Tech football ticket prices increased from \$10 to \$50.

- (a) Ticket prices were \_\_\_\_\_ percent higher in 2010 than they were in 1979.

**Answer:**  $\frac{50-10}{10} = 4 = 400\%$

- (b) On average, ticket prices increased by \_\_\_\_\_ percent annually.

**Answer:**  $(50/10)^{1/31} = 1.053$ , so 5.3% annually

19. Which set of numbers has the higher geometric mean?

- {1, 1, 4, 50}
- {3, 3, 4, 6}

**Answer:**  $(1 \cdot 1 \cdot 4 \cdot 50)^{(1/4)} = 3.76$ , and  $(3 \cdot 3 \cdot 4 \cdot 6)^{(1/4)} = 3.83$

20. If the Alabama has a 23% chance of winning the championship, then what is the probability that they do not win the championship?

**Answer:**  $1 - .23 = .77$

21. Suppose that you look up the CEO's of 50 large companies, and 36 of them are male. You estimate that the probability that the next CEO you look up is male is  $p = \frac{36}{50} = 0.72$ . Is this empirical, subjective, or theoretical probability ?

**Answer:** empirical, based on observational evidence