

Carson-Newman University MATH 201 Test 2, Fall 2018

Directions:

- You may use a stand-alone calculator, but electronic communication is prohibited. No other sources of information are allowed.
- To receive full credit, you must show all relevant work to justify your answer on the test paper. Write down your calculator commands for statistical calculations.
- Clearly identify your final answer, correct to at least 3 significant digits.
- Use notation as described in class.

Honor Pledge: I pledge that I will neither give nor receive unauthorized help on this test from any person, technology, or other resource, and that I will abide by the honor code of Carson-Newman University.

Signed: _____

1. Consider this data for 30-39 year old male drivers that have insured a new mini-van.

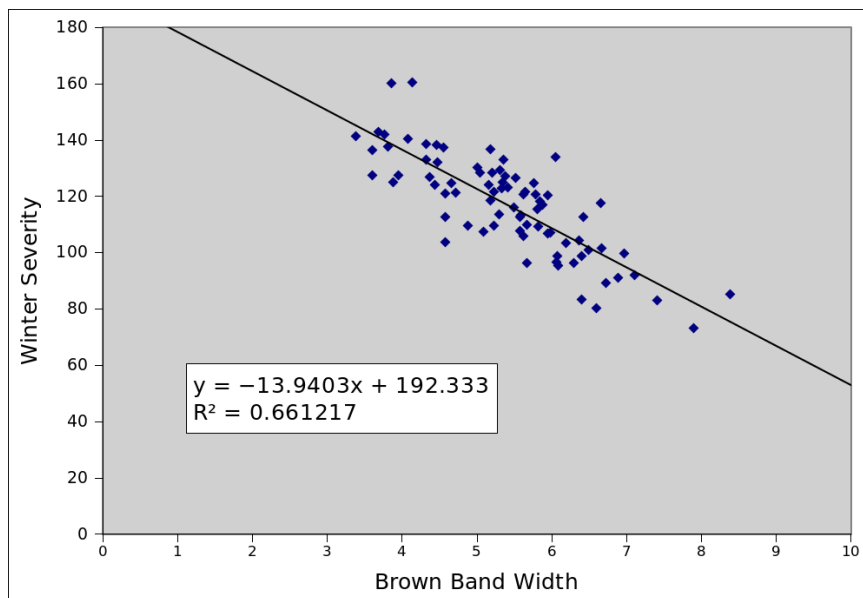
FICO credit score	annual insurance cost
645	1304
662	1142
693	1013
709	995
731	1058
744	1121
760	1036
793	1015
804	968

Let x represent credit score, and y the insurance cost.

- (a) Find the equation of the regression line.
Answer: $y = 2048 - 1.34x$
- (b) Find the correlation between x and y .
Answer: $r = -.71$
- (c) Extrapolate to estimate the insurance cost of someone with a 600 credit score.
Answer: $\hat{y} = 2048 - 1.34(600) = 1244$
2. According to folklore, the narrower the brown band in the middle of a woolly worm, the more severe winter will be.



Suppose a scientist collected this data from the last 80 years



- (a) Find the slope of the linear regression line.
Answer: -13.94
- (b) Find the correlation.
Answer: $r = -\sqrt{.661} = -.813$
3. A roster contains 3 males and 5 females. If you randomly select two distinct names with equal likelihood, find the probability that both are male.
Answer: $(3/8)(2/7) = .107$
4. In a package of 48 widgets, each one independently has an 8% chance of being defective. Let X be the number of defective widgets in the package. Here is the binomial distribution table:

x	P(x)	CDF
0	0.0183	0.0183
1	0.0763	0.0945
2	0.1559	0.2504
3	0.2078	0.4582
4	0.2033	0.6615
5	0.1556	0.8171
6	0.0969	0.9140
7	0.0506	0.9646
8	0.0225	0.9871
9	0.0087	0.9958
10	0.0030	0.9988
11	0.0009	0.9997
12	0.0002	0.9999
13	0.0001	1.0000
14	0.0000	1.0000
15	0.0000	1.0000
16	0.0000	1.0000
17	0.0000	1.0000
18	0.0000	1.0000
19	0.0000	1.0000

(a) Are these probabilities empirical, subjective, or theoretical ?

Answer: $BI(48, .08)$ is theoretical

(b) Find the mean.

Answer: $\mu = (48)(.08) = 3.84$

(c) Find the mode.

Answer: 3

(d) Find the variance.

Answer: $\sigma^2 = (48)(.08)(.92) = 3.53$

(e) Find the 95th percentile.

Answer: $x = 7$

(f) Find the probability that at least 5 widgets are defective.

Answer: $1 - cdf(4) = 1 - .6615 = .3385$

5. A soccer fan is considering his emotional state following this week's big game. Here are the possible outcomes:

	happiness	probability
win	100	.42
lose	-75	.24
draw	-25	

Find his expected happiness after the game.

Answer: $\mu = (100)(.42) + (-75)(.24) + (-25)(.34) = 15.5$

6. The IQ scores of students at an Ivy League school are distributed $N(130, 10)$.
- (a) What is the probability that a randomly selected student has an IQ between 120 and 150 ?
Answer: $normcdf(120, 150, 130, 10) = .819$
- (b) If the freshman class has 1000 students, we expect about 100 of them to have an IQ over _____.
Answer: $invnorm(.9, 130, 10) = 142.8$
7. A flashlight's battery life is $X \sim N(9.3, 1.2)$ hours. Suppose you have two flashlights with fresh batteries, and assume independence.
- (a) Find the probability that a particular flashlight will last more than 10 hours.
Answer: $normcdf(10, 99, 9.3, 1.2) = .28$
- (b) Find the probability that both flashlights will last more than 10 hours.
Answer: $normcdf(10, 99, 9.3, 1.2)^2 = .0783$
- (c) Find the probability that at least one of the flashlights will last more than 10 hours.
Answer: $.28 + .28 - .0783 = 1 - .72^2 = .4815$
8. Consider two fair four-sided dice, each having sides labeled 1,2,3, and 4.
- (a) Let the random variable X be the difference between the higher and lower numbers showing. e.g. if you roll 3 and 1, then $X = 2$. Above each outcome in the sample space, write the value of the random variable.

$$\Omega = \{11, 12, 13, 14, 21, 22, 23, 24, 31, 32, 33, 34, 41, 42, 43, 44\}$$

- (b) Assuming equally likely outcomes, complete the theoretical distribution for X .

X	$P(X)$
0	
1	
2	
3	

X	$P(X)$
0	$4/16 = .25$
1	$6/16 = .375$
2	$4/16 = .25$
3	$2/16 = .125$

Answer:

- (c) Find the expected value of X .
Answer: $\mu = (0)(.25) + (1)(.375) + (2)(.25) + (3)(.125) = 1.25$

9. These college football teams have the given chances of finishing the season undefeated.
- Alabama (74%) Clemson (60%) Notre Dame (43%)
- Assuming independence, find:

- (a) The probability that all three of those teams go undefeated.
Answer: $(.74)(.60)(.43) = .191$
- (b) The probability that none of those teams goes undefeated.
Answer: $(.26)(.40)(.57) = .0593$
- (c) The probability that at least one of those teams goes undefeated.
Answer: $1 - .0593 = .941$
- (d) Exactly one of those teams goes undefeated, i.e. YNN or NYN or NNY
Answer: $(.74)(.40)(.57) + (.26)(.60)(.57) + (.26)(.40)(.43) = .302$
10. A 4G internet provider has $n = 1000$ customers in a certain locality. Let the random variable X be a day's bandwidth usage for a particular customer. The distribution of X has $\mu = 50$ and $\sigma = 70$ megabytes.
- (a) Find the standard error, $\sigma_{\bar{x}}$.
Answer: $70/\sqrt{1000} = 2.21$
- (b) Let \bar{x} be a day's average usage of the 1000 customers. Assume independence. According to the central limit theorem, what distribution models \bar{x} ?
Answer: $\bar{x} \sim N(50, 70/\sqrt{1000})$
- (c) Find the 99th percentile for total megabyte usage in a given day.
Answer: $invnorm(.99, 50, 70/\sqrt{1000}) * 1000 = 55150$
11. Consider two random variables:
- $X \sim N(10, .8)$
 - $Y \sim BI(10, .8)$
- (a) Which one is discrete, X or Y ?
Answer: Y
- (b) Find the mean of X .
Answer: $\mu = 10$
- (c) Find the mean of Y .
Answer: $\mu = (10)(.8) = 8$
- (d) Find $P(X = 11)$
Answer: 0, since X is continuous
- (e) Find $P(Y = 11)$
Answer: 0, since $11 > 10$, this is impossible
12. (Bonus) List up to three things mentioned in class that are positively correlated with the prevalence of dollar stores.
Answer: violence (+), obesity (+), religion (+)