

Test 2 Checklist

1. probability of an event “E”

- (a) $0 \leq P(E) \leq 1$
- (b) the complement of E (“not E”) is denoted \bar{E}

$$P(\bar{E}) = 1 - P(E)$$

2. types of probability:

- (a) empirical: relative frequency of historical results, experiments, simulations
- (b) subjective: educated guess based on the knowledge/belief of the observer
- (c) theoretical: mathematical model, deductive reasoning

3. probability distribution for a random variable x ($x=L1$, $P(x)=L2$)

- (a) sample space Ω is the set of possible outcomes
- (b) for empirical distributions, the sum of frequency is the sample size n
- (c) PDF = probability = proportion = relative frequency
- (d) use CDF to get percentiles
- (e) $\sum P(x) = 1$
- (f) “expected value” is the mean

4. randomness, simulations, law of large numbers

5. equally likely outcomes $P(E) = \frac{\# E}{\# \Omega}$

6. joint probability

- (a) “not” - do the complement
 $P(\text{not } A) = 1 - P(A)$
- (b) “and” - multiply probabilities of each step, given the previous ones
 $P(A \text{ and } B) = P(A)P(B|A)$
- (c) A and B are independent if $P(A \text{ and } B) = P(A)P(B)$ (the events don’t inform each other)
- (d) with vs without replacement
- (e) “or” - use addition, but don’t double-count if the events are not disjoint (Venn diagram)
 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
- (f) if an event could happen different ways, add the probabilities
e.g. probability of 2 heads in three tosses = $P(\text{HHT}) + P(\text{HTH}) + P(\text{THH})$
- (g) “at least one” easier to work with if you translate to “not zero”

7. binomial distribution $X \sim BI(n, p)$

- (a) x is the sum of repeated, independent, and identical binary trials
- (b) theoretical mean $\mu = np$, st dev $\sigma = \sqrt{np(1-p)}$
- (c) $\text{binompdf}(n, p, x)$ gives the probability of exactly x
- (d) $\text{binomcdf}(n, p, x)$ gives the probability of less than or equal to x
- (e) be able to recognize binomial distribution in applications

8. normal distribution $X \sim N(\mu, \sigma)$

- (a) bell shaped with thin tails
- (b) 68, 95, 99.7 “empirical rule”, and connection with z-scores
- (c) draw a picture when you work these problems
- (d) $\text{normcdf}(a, b, \mu, \sigma)$ gives the probability that x is between a and b
- (e) $\text{invnorm}(A, \mu, \sigma)$ gives the x value with area A to the left of it

9. sampling distribution

- (a) empirical/sample statistics (\bar{x}, s) versus theoretical/population statistics (μ, σ) .
- (b) \bar{x} (the group average) is itself a random variable, and

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

this is called the “standard error” of \bar{x}

- (c) Central Limit Theorem: if the individuals are independent, then as n gets larger,

$$\bar{x} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

- (d) use the CLT when you are interested in the sample/group mean or total

10. Linear regression and correlation

- (a) scatterplots
- (b) use LinRegTtest to find and work with the regression line $y = a + bx$ that best fits the data.
- (c) the slope of the line is the coefficient of the x .
- (d) You can use the regression line to estimate y for a given x :
 - interpolation if x is within known points
 - extrapolation if it lies outside known points
- (e) the sample correlation r is between -1 and 1
- (f) $|r|$ measures how tightly the cloud of data points cluster around the regression line.
- (g) r has the same sign as the slope
- (h) Be able to explain positive/negative and weak/strong correlation intuitively.
- (i) Know the difference between correlation and causation.

11. models

- (a) be able to select an appropriate model for the given problem or data
- (b) discrete (e.g. binomial) vs continuous (e.g. normal) distributions
- (c) area under the graph represents probability
- (d) for a continuous random variable and a number c , $P(x = c) = 0$; because a line contains zero area
- (e) translate words into equations or inequalities, and then find probability using PDF or CDF
- (f) be able to work two step problems
- (g) be able to solve for parameters (e.g. n, p, μ, σ) to satisfy a given requirement, possibly using guess-check
- (h) remember that many models assume that observations are independent