MATH 208 Test 3, Spring 2020

Directions:

- This test is open book. You may use any resource linked to from the class webpage.

- Do not seek help from any other individual, whether in person or electronically.

- You may use Octave to verify calculations.
  These demos might be useful `https://massey.limfinity.com/208/sage.htm`

- Use notation conventions as described in class.

- To receive full credit, you must **show all relevant work to completely justify your answer.**

- You have 24 hours to email me a photo of your work. Organize your work clearly.

- 105 points possible, graded out of 100 points.

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1. (24 pts) Write out the entries of 2×2 matrices that do the following linear transformations. Use decimals, rounded to the nearest .001.

   (a) Write R to rotate clockwise 10 degrees.

   (b) Write D to scale by a factor of 5 horizontally, and a factor of 2 vertically.

   (c) Write P to project orthogonally onto the line $y = \frac{x}{6}$.

   (d) Write S to shear horizontally by a factor of 3

   (e) Write Y to reflect across the y-axis.

   (f) Write F to flatten to 40% around the line $y = \frac{x}{6}$, i.e. $F = .6P + .4I$

2. (18 pts) Find the inverse of each of the matrices (if possible).

   (a) R^{-1}

   (b) D^{-1}

   (c) P^{-1}

   (d) S^{-1}

   (e) Y^{-1}

   (f) F^{-1}

3. (15 pts) Does the pair of transformations commute ? Yes or no?

   (a) R and D (d) R and Y (g) D and S (j) P and S (m) S and Y

   (b) R and P (e) R and F (h) D and Y (k) P and Y (n) S and F

   (c) R and S (f) D and P (i) D and F (l) P and F (o) Y and F

4. (16 pts) Let A be a 3×2 matrix with entries given by $a_{ij} = i(i+j)$.

    (a) Write the matrix A.

    (b) Write the matrix $B = A^T A$.

    (c) Find $\det(B)$.

    (d) Find B^{-1}.

5. (16 pts) The matrix $P = \frac{1}{325} \begin{bmatrix} 289 & -102 \\ -102 & 36 \end{bmatrix}$ does an orthgonal projection onto a vector v.

    (a) Find the vector v.

    (b) Find the matrix R that orthogonally reflects across v.

    (c) Let $x = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$.

        i. Find $\|x\|$.

        ii. Find $\|Px\|$.

        iii. Find $\|Rx\|$.

    (d) Find the angle between v and x.

6. (6 pts) Let $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$. If $AMB = A^2$, then find the matrix M.

7. (10 pts) Let $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$.

    (a) Find matrices L (lower triangular) and U (upper triangular) so that $A = LU$.

    (b) Find matrices Q (with $Q^T Q = I$) and R (upper triangular) so that $A = QR$.