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MATH 208 Test 4, Spring 2020

Directions:

- This test is open book. You may use any resource linked to from the class webpage.

- You must work alone. Do not seek help from any other individual, whether in person or electronically.

- You may use Octave to check your answers, but all work should be done "by hand".

- Use notation conventions as described in class.

- To receive full credit, you must **show all relevant work to completely justify your answer.**

- You have until Tue, Apr 21 at 8am Jeff City time to email me your work. Organize your work clearly.

- 105 points possible, graded out of 100 points.

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1. (12 pts) Find the area of the quadrilateral with corners at $(0,0)$, $(5,2)$, $(6,7)$, and $(3,9)$.
   Include a sketch as you show your work.

2. (10 pts) Find a vector w, with $\|w\| = 100$, such that w is orthogonal to both $u = \begin{bmatrix} 7 \\ 2 \\ 1 \end{bmatrix}$ and $v = \begin{bmatrix} 5 \\ 9 \\ 3 \end{bmatrix}$.

3. (18 pts) By hand: Find the eigenvalues of this matrix. And for each eigenvalue, find an eigenvector with integer entries.
$$A = \begin{bmatrix} 5 & 0 & 4 \\ 0 & 2 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

4. (12 pts) Let $A = \begin{bmatrix} 5 & x \\ x & 2 \end{bmatrix}$ be a symmetric matrix. If the largest eigenvalue is 10, find $\det(A)$.

5. (10 pts) Let A and B be 5×5 matrices. If $\det(A) = 20$ and $\det(B) = 8$, find $\det(-2A^{-1}B^3)$.

6. (15 pts) Create a matrix that has these eigenpairs:
$$\left\{ \left(5, \begin{bmatrix} 4 \\ 11 \end{bmatrix}\right), \left(-2, \begin{bmatrix} 3 \\ 8 \end{bmatrix}\right) \right\}$$

7. (10 pts) Suppose A is 3×3.
   If $\det(A) = 70$, the trace of A is 2, and $A - 8I$ is singular, then find the eigenvalues of A.

8. Let $A = \begin{bmatrix} x & 1 & 0 & 0 \\ 1 & x & 1 & 0 \\ 0 & 1 & x & 1 \\ 0 & 0 & 1 & x \end{bmatrix}$, where $x \geq 0$.

   (a) (6 pts) If $x = 10$, find $\det(A)$. Show your work to calculate the determinant by hand.

   (b) (12 pts) If $\det(A) = 10$, find x.
       (Full credit if you find x EXACTLY, $\frac{1}{2}$ credit if you approximate it to the nearest 0.1.)