NAME ____________________________

MATH 208 Final Exam, Spring 2020

Directions:

- This test is open book. You may use any resource linked to from the class webpage.
- You must work alone. Do not seek help from any other individual, whether in person or electronically.
- To receive full credit, you must **show all relevant work to completely justify your answer.**
- Please use Octave to check your answers, but all work should be done "by hand", needing only a handheld calculator.
- Use notation conventions as described in class.
- You have until Sat, Apr 25 at 8am Jeff City time to email me your work. Organize your work clearly.
- Each of the 12 questions is worth 9 points. So 108 points are possible; graded out of 100 points.

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1. Let $f(a,b)$ be a linear function giving the cost of a apples and b bananas.
   If $f(3,1) = 2.33$ and $f(1,3) = 1.79$, then evaluate $f(2,8)$.

2. Here is an augmented matrix:
   $$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 0 & 2 & 3 \\ 2 & 7 & 4 & 9 \end{array}\right]$$
   Perform the next two row operations of Gaussian elimination.

3. Find the 2×2 matrix P that orthogonally projects vectors onto the line $y = 0.2x$.

4. Let A be a 5×2 matrix with entries given by $a_{ij} = ij - 1$.
   By hand, show how to compute $(A^T A)^{-1}$.

5. Let $A = \begin{bmatrix} 8 & 21 \\ 12 & 9 \end{bmatrix}$. The two columns sum to 20 and 30 respectively.
   There is a diagonal matrix D so that each column of AD sums to 100.
   Find D, and then compute DA.

6. Let $u = \begin{bmatrix} 2 \\ 4 \\ 9 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ 5 \\ v_3 \end{bmatrix}$, where $v_3 > 5$.

   (a) If $v_3 = 20$, find the angle (to the nearest degree) between u and v.

   (b) If the angle between u and v is $20°$, find v_3 to the nearest integer.
   You may set up an equation to solve, or use trial and error.

7. Let $Q = \begin{bmatrix} .352 & -.936 \\ .936 & .352 \end{bmatrix}$ be a rotation matrix.

   (a) Multiplying by Q rotates a vector by _______ degrees.

   (b) Find a matrix S such that $S^2 = M$.

8. Suppose $\text{rref}(A+5I) = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -0.8 \\ 0 & 0 & 0 \end{bmatrix}$.
   Find an eigenpair (λ, x) so that $Ax = \lambda x$, and x has integer entries.

9. Let $A = \begin{bmatrix} 15 & -3 \\ 2 & 8 \end{bmatrix}$
   By hand, diagonalize A (i.e. find a matrix V and diagonal matrix Λ such that $A = V\Lambda V^{-1}$)

10. Make up a 4×4 upper triangular matrix that has these properties:
    - determinant 210
    - trace 17
    - $\lambda = 2$ is an eigenvalue
    - $A \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ 10 \\ 5 \end{bmatrix}$

11. If $\lambda = 6$ is an eigenvalue of A, compute the value of:
    $$\frac{x^T A x}{x^T A^{-1} x}$$

12. Let $A = \begin{bmatrix} a_{11} & 26 & 13 \\ -31 & 52 & 23 \\ 36 & -56 & a_{33} \end{bmatrix}$. You are given that $x = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$ is an eigenvector of A.
    Find the associated eigenvalue, and then find a_{11} and a_{33}.