NAME ____________________________

MATH 208 Test 1, Spring 2020

Directions:

- Do not use any notes, books, the internet, or other sources of information.
- You may use a calculator for arithmetic calculations.
- Use notation conventions as described in class.
- You have 55 minutes. You must work alone; do not communicate with any other person.
- To receive full credit, you must **show all relevant work to completely justify your answer.**
- 105 points possible, graded out of 100 points.

---

1. (6 pts) Let $f(a, b)$ be a linear function giving the time it takes for an elf to assemble a airplanes and b bulldozers. If $f(3, 0) = 63$ and $f(0, 5) = 65$, then evaluate $f(2, 8)$.
   Answer: takes 21 minutes and 13 minutes for each toy respectively, so $f(2, 8) = 2(21) + 8(13) = 146$ minutes

2. (12 pts) Solve the linear systems for x:

   (a) $\begin{bmatrix} 4 & 2 \\ 0 & 5 \end{bmatrix} x = \begin{bmatrix} 26 \\ 30 \end{bmatrix}$

   Answer: $x = \begin{bmatrix} 7/2 \\ 6 \end{bmatrix}$

   (b) $\begin{bmatrix} 2 & 0 & 4 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix} x = \begin{bmatrix} 0 \\ 12 \\ 9 \end{bmatrix}$

   Answer: $x = \begin{bmatrix} -3 \\ 4 \\ 1.5 \end{bmatrix}$

3. (6 pts) Let v, w be unit vectors. If $\|v + w\|^2 = \frac{5}{2}$, find $v \cdot w$.
   Answer: $5/2 = (v + w) \cdot (v + w) = 1 + 1 + 2v \cdot w$, so $v \cdot w = 0.25$

4. (25 pts) Consider this matrix, which represents 3 smoothie recipes using 5 types of fruit.

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 4 \\ 5 & 8 & 0 \\ 7 & 3 & 5 \\ 2 & 5 & 8 \end{bmatrix}$$

   (a) This is a ____ $\times$ ____ matrix.
   Answer: 5×3

   (b) Find the indices for which $a_{ij} = 3$.
   Answer: $a_{21} = a_{13} = a_{42} = 3$

   (c) Compute $a_1 \cdot a_3$
   Answer: 1st and 3rd columns, 69

(d) Compute $a_{1:} \cdot a_{3:}$
Answer: 1st and 3rd rows, 18

(e) Write A^T.
Answer: flip rows and columns

(f) Compute $a_1 + 2a_3$.
Answer: $[8; 11; 5; 17; 18]$

(g) Let $x = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$. Compute Ax.
Answer: same thing, $[8; 11; 5; 17; 18]$

(h) Let $\vec{1} \in \mathbb{R}^5$ be a vector of all ones. Compute $\vec{1} \cdot (Ax)$.
Answer: $[1; 1; 1; 1; 1] \cdot [8; 11; 5; 17; 18] = 59$

(i) Let $p \in \mathbb{R}^5$ be a vector of prices for each fruit. If $p \cdot a_1 = 3.60$ and $p \cdot a_2 = 4.20$, compute $p \cdot (2a_1 + 5a_2)$.
Answer: 2 of the first smoothie and 5 of the second will cost $2(3.60) + 5(4.20) = 28.2$

5. (10 pts) The equations $x_1 + 2x_2 = 50$ and $2x_1 - 3x_2 = 16$ describe intersecting lines.

(a) Find $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, which represents the point of intersection.
Answer: $x = [26; 12]$

(b) Express the system of equations in the form $Ax = b$.
Answer: $\begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 26 \\ 12 \end{bmatrix} = \begin{bmatrix} 50 \\ 16 \end{bmatrix}$

6. (18 pts) At the market,

- Xerxes bought 2 apples and 6 bananas.
- Yolanda bought 5 apples and 3 bananas.

[E1] An apple costs 50¢ more than a banana.

[E2] Yolanda's fruit purchase was 50% more expensive than Xerxes'.

Let x_1 be the price of an apple, and x_2 the price of a banana.

(a) Write [E1] as an equation involving x_1 and x_2.
Answer: $x_1 - x_2 = 50$

(b) Write [E2] as an equation involving x_1 and x_2.
Answer: $5x_1 + 3x_2 = 1.5(2x_1 + 6x_2)$

(c) Rearrange those equations to express a matrix-vector product: $Ax = b$.
Answer: $\begin{bmatrix} 1 & -1 \\ 2 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \end{bmatrix}$

(d) Find the prices x_1 and x_2.
Answer: $x = [75; 25]$

7. (15 pts) Let $v = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$ and $w = \begin{bmatrix} 5 \\ -3 \\ w_3 \end{bmatrix}$ be two vectors.

(a) Find a scalar α so that $\|\alpha v\| = 3$.
Answer: $\frac{3}{\sqrt{54}}$

(b) If $v \perp w$, find w_3.
Answer: $v \cdot w = 5 - 21 + 2w_3 = 0$, so $w_3 = 8$

(c) Find $\|v + w\|$.
Answer: by Pythag.Thm, $\sqrt{v \cdot v + w \cdot w} = \sqrt{54 + 98} = \sqrt{152}$, or $\sqrt{6^2 + 4^2 + 10^2} = \sqrt{152}$

8. (13 pts) In one blender you have made a smoothie that is 25% apple and 75% banana. In another blender you have a smoothie made of 60% apple and 40% banana. Let the columns of $A = \begin{bmatrix} .25 & .60 \\ .75 & .40 \end{bmatrix}$ represent those two blends.

(a) Pour 8 oz from the first blender and 2 oz from the second blender into your glass.
Compute $8a_1 + 2a_2$. What percentage of that concoction is apple ?
Answer: $\begin{bmatrix} 3.2 \\ 6.8 \end{bmatrix}$ is 32% apple

(b) Find a linear combination of a_1 and a_2 that produces a mixture with 5 oz of apple and 5 oz of banana. i.e. solve $Ax = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$.
Answer: $x = [20/7, 50/7]$

(c) If you pour 8 oz from the first blender, how much from the second blender would bring the mixture to 50% apple ?
Answer: set the amount of apple and banana equal to each other: $(.25)(8) + .6c = (.75)(8) + .4c$; solve to get $c = 20$, or from previous part - notice that you need 5/2 times as much from the 2nd blender