NAME _____

MATH 208 Test 3, Spring 2020

Directions:

- This test is open book. You may use any resource linked to from the class webpage.
- Do not seek help from any other individual, whether in person or electronically.
- You may use Octave to verify calculations.
  These demos might be useful `https://massey.limfinity.com/208/sage.htm`
- Use notation conventions as described in class.
- To receive full credit, you must **show all relevant work to completely justify your answer.**
- You have 24 hours to email me a photo of your work. Organize your work clearly.
- 105 points possible, graded out of 100 points.

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1. (24 pts) Write out the entries of 2×2 matrices that do the following linear transformations. Use decimals, rounded to the nearest .001.

   (a) Write R to rotate clockwise 10 degrees.

   Answer: $\theta = -10$ degrees, $R = \begin{bmatrix} .985 & .174 \\ -.174 & .985 \end{bmatrix}$

   (b) Write D to scale by a factor of 5 horizontally, and a factor of 2 vertically.

   Answer: $D = \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$

   (c) Write P to project orthogonally onto the line $y = \frac{x}{6}$.

   Answer: with $v = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$, $P = \frac{vv^T}{v^T v} = \begin{bmatrix} .973 & .162 \\ .162 & .027 \end{bmatrix}$

   (d) Write S to shear horizontally by a factor of 3

   Answer: $S = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

   (e) Write Y to reflect across the y-axis.

   Answer: $Y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

   (f) Write F to flatten to 40% around the line $y = \frac{x}{6}$, i.e. $F = .6P + .4I$

   Answer: $F = \begin{bmatrix} .984 & .097 \\ .097 & .416 \end{bmatrix}$

2. (18 pts) Find the inverse of each of the matrices (if possible).

   (a) R^{-1}

   Answer: $R^{-1} = \begin{bmatrix} .985 & -.174 \\ .174 & .985 \end{bmatrix}$

   (b) D^{-1}

   Answer: $D^{-1} = \begin{bmatrix} 1/5 & 0 \\ 0 & 1/2 \end{bmatrix}$

(c) P^{-1}
Answer: $\det(P) = 0$, so no inverse

(d) S^{-1}
Answer: $S^{-1} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$

(e) Y^{-1}
Answer: $Y^{-1} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

(f) F^{-1}
Answer: $F = -1.5P + 2.5I = \begin{bmatrix} 1.041 & -.243 \\ -.243 & 2.459 \end{bmatrix}$

3. (15 pts) Does the pair of transformations commute ? Yes or no?
Answer: You can check with Octave, or think about the transformations geometrically.

(a) R and D
Answer: no

(b) R and P
Answer: no

(c) R and S
Answer: no

(d) R and Y
Answer: no

(e) R and F
Answer: no

(f) D and P
Answer: no

(g) D and S
Answer: no

(h) D and Y
Answer: yes

(i) D and F
Answer: no

(j) P and S
Answer: no

(k) P and Y
Answer: no

(l) P and F
Answer: yes

(m) S and Y
Answer: no

(n) S and F
Answer: no

(o) Y and F
Answer: no

4. (16 pts) Let A be a 3×2 matrix with entries given by $a_{ij} = i(i+j)$.

   (a) Write the matrix A.

   Answer: $A = \begin{bmatrix} 2 & 3 \\ 6 & 8 \\ 12 & 15 \end{bmatrix}$

   (b) Write the matrix $B = A^T A$.

   Answer: $B = \begin{bmatrix} 184 & 234 \\ 234 & 298 \end{bmatrix}$

   (c) Find $\det(B)$.

   Answer: 76

   (d) Find B^{-1}.

   Answer: $\frac{1}{76} \begin{bmatrix} 298 & -234 \\ -234 & 184 \end{bmatrix}$

5. (16 pts) The matrix $P = \frac{1}{325} \begin{bmatrix} 289 & -102 \\ -102 & 36 \end{bmatrix}$ does an orthgonal projection onto a vector v.

   (a) Find the vector v.

   Answer: Fit the pattern that $v_1^2 = 289$ and $v_1 v_2 = -102$ to get $v = \begin{bmatrix} 17 \\ -6 \end{bmatrix}$

   (b) Find the matrix R that orthogonally reflects across v.

   Answer: $R = 2P - I = \begin{bmatrix} .778 & -.628 \\ -.628 & -.778 \end{bmatrix}$

   (c) Let $x = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$.

   i. Find $\|x\|$.
   Answer: 10

   ii. Find $\|Px\|$.
   Answer: 5.547

   iii. Find $\|Rx\|$.
   Answer: 10

   (d) Find the angle between v and x.

   Answer: $\theta = \cos^{-1}(\frac{v^T x}{\|v\|\|x\|}) = 56.3°$

6. (6 pts) Let $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$. If $AMB = A^2$, then find the matrix M.

   Answer: pre-multiply both sides by A^{-1} and post-multiply by B^{-1}

   $M = A^{-1}A^2B^{-1} = AB^{-1} = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ -15 & 11 \end{bmatrix}$

7. (10 pts) Let $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$.

   (a) Find matrices L (lower triangular) and U (upper triangular) so that $A = LU$.

   Answer: $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

   (b) Find matrices Q (with $Q^T Q = I$) and R (upper triangular) so that $A = QR$.

   Answer: Q needs to rotate by $\tan^{-1}(2/1)$, then let $R = Q^T A$.

   $A = \begin{bmatrix} .447 & -.894 \\ .894 & .447 \end{bmatrix} \begin{bmatrix} 2.24 & 3.13 \\ 0 & .447 \end{bmatrix}$