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MATH 313 Final Exam, Spring 2019

Directions:

- To receive full credit, you must **show all relevant work to completely justify your answer**.
- Use notation conventions from class.
- This exam is open book/notes.
- You may use a calculator, but NOT the internet.
- You must work alone; do not communicate with anybody else in any way.

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1. At a certain point in time, $\vec{v} = \begin{bmatrix} 2 \\ -6 \\ 9 \end{bmatrix}$ and $\vec{a} = \begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix}$. Find the radius of curvature.

2. Let $\vec{F} = \begin{bmatrix} 2x - y + z \\ xy + z \\ xz^2 \end{bmatrix}$ be a vector field.

   (a) Compute the divergence $\nabla \cdot \vec{F}$.

   (b) Compute the curl $\nabla \times \vec{F}$.

3. A infinite series $\sum_{k=0}^{\infty} a_k$ has partial sums with

$$s_n = \frac{4n(6n+2)}{3(n+1)^2}$$

   (a) Find a_5.

   (b) Find the value of the infinite series.

4. Find the sum of the geometric series $60 - 50 + \cdots$.

5. A fourth degree polynomial has $f(5) = 7$, $f'(5) = 20$, $f''(5) = -18$, $f'''(5) = 30$, and $f^{(4)}(5) = 56$. Find $f(2)$.

6. A function $f(x, y)$ has a local minimum at $(2, 5)$. If at that point, $f_{xx} = 8$ and $\nabla^2 f = 13$, find the largest possible integer value of f_{xy}. (Hint: look at the Hessian matrix)

7. You want to find the point $P(x, y, z)$ on the surface of $z = x^2 y$ that is closest to the point $Q(1, 2, 9)$.

   (a) Write an objective function $f(x, y, z)$ that represents the squared distance between P and Q.

   (b) Write (but do not solve) the Lagrange system of equations to solve this constrained optimization problem.

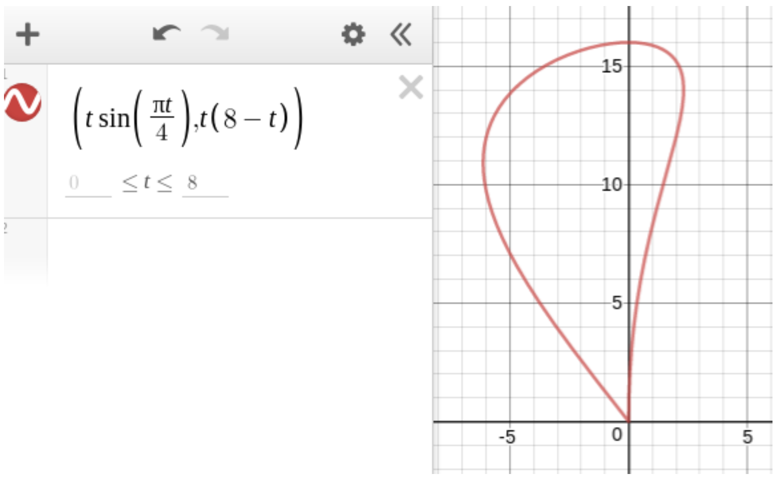
8. Let C be the curve described by

$$x(t) = 5 + \cos(\pi t) \qquad y(t) = 1 + 2^t \qquad t \in [0, 3]$$

   Evaluate $\int_C (2x/y + 2y^3)dx + (6y^2 x - x^2/y^2)dy$.
   (Verify the field is conservative and utilize the potential function.)

9. An object is accelerating with $\vec{a}(t) = \begin{bmatrix} e^{-t/8} \\ \frac{t}{10}(8-t) \end{bmatrix}$. If the initial velocity is $\vec{v}(0) = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$, find the object's speed when $t = 8$.

10. A wall surrounds the city pictured here (with x, y in km).



The base of the wall lies in the xy-plane, and the height of the wall is given by $h(x, y) = 100 - xy$ meters.

(a) Write an integral to compute the surface area of the wall in square meters.

(b) Write an integral to compute the area of the city in square km.

11. Consider this triple integral:

$$\int_0^{\pi} \int_0^{1+\theta} \int_0^{2\sqrt{r}} z^2 r^4 \, dz \, dr \, d\theta$$

(a) Evaluate it by hand, showing all steps.

(b) If the integral represents the moment M_{xy} of a solid described in cylindrical coordinates, find the density of the solid at the point $x = -3, y = 1, z = 2$.