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MATH 211 Test 1, Fall 2019

Directions:

- Do not use any notes, books, the internet, or other sources of information.
- You may use a calculator for arithmetic calculations.
- You have 55 minutes. You must work alone; do not communicate with any other person.
- To receive full credit, you must **show all relevant work to completely justify your answer (on separate paper)**.
- 106 points possible, graded out of 100 points.

Formulas

- $\displaystyle\int_a^b \frac{1}{2} r^2 d\theta$
- $\displaystyle\int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$
- $\displaystyle\left|\int_a^b y dx\right|$
- $\displaystyle\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$

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1. (14 pts) When $t = 30$, suppose $x = -75$, $y = 42$, $\frac{dx}{dt} = -15$, and $\frac{dy}{dt} = 10$,

   (a) Find polar coordinates for the position $(-75, 42)_C$.
   Answer: $(2.63, \sqrt{7389})_P$

   (b) Estimate the Cartesian position 2 seconds earlier, i.e. $x(28)$ and $y(28)$.
   Answer: $x(28) \approx -75 - 15(-2) = -45$ and $y(28) \approx 42 + 10(-2) = 22$

2. (22 pts) Parameterize the following for $t \in [0, 60]$ minutes.

   (a) a line that starts at $(2, 16)$ when $t = 0$, and terminates at $(9, 4)$ when $t = 60$.
   Answer: $x(t) = 2 + \frac{7}{60}t$ and $y(t) = 16 - \frac{12}{60}t$

   (b) a circle with area 36π, centered at $(9, 4)$;
   starts at the western-most point when $t = 0$ moving clockwise with a period of 60.
   Answer: $x(t) = -6\cos(\frac{\pi}{30}t) + 9$ and $y(t) = 6\sin(\frac{\pi}{30}t) + 4$

3. (10 pts) Set up the integral to find the length of $y = \frac{1}{x^2}$ for $x \in [1, 3]$.
   Answer: parameterized $(t, 1/t^2)$ you get $\frac{dy}{dt} = -2/t^3$, so $L = \int_1^3 \sqrt{1 + 4t^{-6}} dt$

4. (30 pts) Consider the parametric equations:

$$x(t) = 7t - 2t^2 \qquad y(t) = t^2 \qquad t \in [0, 4]$$

   (a) Make a table of points and sketch the graph.
   Answer: $(0, 0, 0), (1, 5, 1), (2, 6, 4), (3, 3, 9), (4, -4, 16)$

   (b) Find the equation of the tangent line at the point when $t = 2$.
   Answer: $\frac{dx}{dt} = 7 - 4t$ and $\frac{dy}{dt} = 2t$, so slope is $\frac{4}{-1} = -4$, and tan.line is $y = 4 - 4(x - 6)$

   (c) Find the area enclosed between the parametric curve and the y-axis (set up only)
   Answer: crosses when $t = 7/2$, so $A = \left|\int_0^{3.5} t^2(7 - 4t) dt\right|$

5. (30 pts) R is the region inside the loop made by the polar graph $r = 6\theta^2(1-\theta)$ for $\theta \in [0,1]$ radians.

(a) Find the perimeter of R (set up integral).
Answer: $\int_0^1 \sqrt{(6\theta^2(1-\theta))^2 + (12\theta - 18\theta^2)^2} d\theta$

(b) Find the area of R (evaluate the integral by hand).
Answer: $18 \int_0^1 \theta^4(1-\theta)^2 d\theta = \int_0^1 \theta^4 - 2\theta^5 + \theta^6 d\theta = 6/35$

(c) Using calculus, find the Cartesian coordinates of the point on the loop that is furthest from the origin.
Answer: $\frac{dr}{d\theta} = 12\theta - 18\theta^2 = 0$ where $\theta = 2/3$ and $r = 8/9$, so $(.70, .55)_C$