NAME ____________________________

## MATH 211 Test 1, Fall 2018

**Directions:**

- You may use a calculator for arithmetic calculations.
- Do not use any notes, books, the internet, or other sources of information.
- You must work alone; do not communicate with any other person.
- To receive full credit, you must **show all relevant work to completely justify your answer (on separate paper)**.
- Use notation conventions from class.
- Use radians unless otherwise instructed.
- 55 points possible, graded out of 50 points.

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## Formulas

- $\int_a^b \frac{1}{2} r^2 d\theta$
- $\int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$
- period $= \frac{2\pi}{\omega}$

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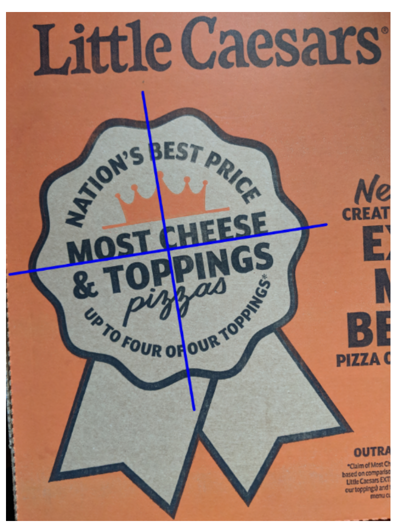
1. (5 pts) Give two polar coordinate aliases for the Cartesian point $(-12, 5)_C$.
   **Answer:** $r = \sqrt{12^2 + 5^2} = 13$ and $\theta = \tan^{-1}(5/-12) = -.395$, but the point is in the 2nd quadrant, so $(2.75, 13)_P$, $(-.395, -13)_P$

2. (5 pts) Little Caesar's sells a 14 inch diameter pizza. What if they put waves in the crust so that it would look like the seal on their pizza box ?



Suppose $(0, 7.4)_P$ and $(\frac{\pi}{2}, 6.6)_P$ are points on the boundary furthest from and closest to the center respectively.

Find a polar equation that describes the wavy boundary.
**Answer:** $r = 7 + .4\cos(10\theta)$

3. (5 pts) Write the integral to compute the area of the polar region bounded by $r = \theta e^{-\theta}$ and $|\theta - \frac{\pi}{2}| = \frac{\pi}{6}$, and above the $x$-axis.

   **Answer:** the angles are between $\frac{\pi}{2} \pm \frac{\pi}{6}$, so $A = \dfrac{1}{2}\displaystyle\int_{\frac{2\pi}{6}}^{\frac{4\pi}{6}} (\theta e^{-\theta})^2 d\theta$

4. (10 pts) Consider the region bounded by the line $y = x$ and this parametric curve:

$$x(t) = t^2 - 7t + 6$$

$$y(t) = 2t^2 - 16t + 20$$

   (a) Draw a sketch with the intersection coordinates labeled. (Hint: find the intersection times first.)
   **Answer:** intersects if $x(t) = y(t)$, so $t^2 - 9t + 14 = (t-2)(t-7) = 0$ so $t = 2, 7$; plug in to get $(x, y)$ coordinates $(-4, -4)$ and $(6, 6)$, another point $(-6, -12)$ when $t = 4$, so it loops below the diagonal line

   (b) Find the coordinates of the western-most point in the region.
   **Answer:** $x$ is minimized when $\frac{dx}{dt} = 2t - 7 = 0$, so $t = 3.5$, so $(-6.25, -11.5)$

   (c) Find the perimeter. (do not evaluate the integral)
   **Answer:** $\sqrt{10^2 + 10^2} + \int_2^7 \sqrt{(2t-7)^2 + (4t-16)^2} dt$.

5. (6 pts) At a certain point on a parametric curve, the tangent line is $y = 5 + 0.38(x - 13)$.

   (a) If at that point $\frac{dy}{dt} = 7$, find $\frac{dx}{dt}$.
   **Answer:** $\frac{dy}{dx}\frac{dx}{dt} = \frac{dy}{dt}$, so $.38\frac{dx}{dt} = 7$ and $\frac{dx}{dt} = 18.42$

   (b) If $x(9) = 13$, use the differential to estimate $x(9.04)$.
   **Answer:** $x(9) + \frac{dx}{dt}(9.04 - 9) = 13 + 18.42(.04) = 13.74$

6. (6 pts) Parameterize a circle for $t \geq 0$ such that

   - frequency 20 RPM
   - circumference $12\pi$
   - center $(10, 0)$
   - starts at bottom and moves counter-clockwise

   **Answer:** $\omega = 2\pi(20) = 40\pi$, and $2\pi R = 12\pi$, so $R = 6$. We want to start at the bottom, with $x$ increasing, so
   $x = 10 + 6\sin(40\pi t)$ and $y = -6\cos(40\pi t)$

7. (6 pts) Suppose a parametric curve is traced by $(x(t), y(t))$ for $t \in [0, 60]$. Consider modifications of the form: $(x(at) + b, c\, y(at))$.

   (a) What effect does $a = 4$ have ?
   **Answer:** makes it go 4 times as fast, finishing for $t \in [0, 15]$

   (b) What effect does $b = 10$ have ?
   **Answer:** shifts the graph right ten units

(c) What effect does $c = 2$ have ?
**Answer:** stretches the graph vertically by a factor of 2

8. (5 pts) Suppose a parametric curve traces a closed curve for $t \in [a, b]$. If $\frac{dx}{dt} = 4\cos(t)$ and $\frac{dy}{dt} = t^2 - 5$, write the integral to compute the enclosed area. (You don't have to find $a$ and $b$.)
**Answer:** $\int_a^b y dx = \int_a^b y \frac{dx}{dt} dt = \int_a^b (\frac{1}{3}t^3 - 5t) 4\cos(t) dt$

9. (7 pts) Suppose I have a polar function $r(\theta)$. At a certain point on the graph, in the second quadrant: $\sin(\theta) = .8$, $r = 20$, and $\frac{dr}{d\theta} = -5$ Find the equation of the tangent line at that point.
**Answer:** in the 2nd quadrant, $\cos(\theta) = -.6$, so the point is $(-12, 16)$. The slope is $\frac{dy}{dx} = \frac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)} = \frac{\frac{dr}{d\theta}\sin\theta + r(\cos\theta)}{\frac{dr}{d\theta}\cos\theta + r(-\sin\theta)} = \frac{(-5)(.8)+20(-.6)}{(-5)(-.6)-20(.8)} = \frac{-16}{-13}$. So the tangent line is $y = 16 + \frac{16}{13}(x + 12)$