NAME ____________________________

MATH 211 Test 2, Fall 2018

Directions:

- You may use a calculator for arithmetic calculations.
- Do not use any notes, books, the internet, or other sources of information.
- You must work alone; do not communicate with any other person.
- To receive full credit, you must **show all relevant work to completely justify your answer (on separate paper)**.
- Use notation conventions from class.
- 55 points possible, graded out of 50 points.

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1. (4 pts) Let $P(2,9,-8)$ and $Q(18,21,13)$. Find a vector that points from P to Q, and then another 5 units beyond.
   Answer: The length of $\vec{PQ}$ is 29, so stretch it to have length 34: $\frac{34}{29}[16;12;21]$

2. (4 pts) Find a value of c such that $[6,c,-4]$ and $[\frac{1}{c},1,5]$ are orthogonal vectors.
   Answer: solve $6/c+c-20=0$ to get $c=10\pm\sqrt{94}\approx 19.695, .305$

3. (4 pts) Find the distance from the point $P(2,9,-8)$ to the sphere $x^2+y^2+z^2=6x-10z$.
   Answer: complete the squares to get: $(x-3)^2+y^2+(z+5)^2=34$, so the center is $(3,0,-5)$, which is $\sqrt{91}$ from P. So P is $\sqrt{91}-\sqrt{34}\approx 3.71$ units from the sphere

4. (12 pts) Let $P(2,9,-8)$, $Q(18,21,13)$, $R(7,6,2)$, and $S(5,10,15)$ be four points in $\mathbb{R}^3$.

   (a) Find the equation of the plane containing P, Q, and R.
   Answer: $\vec{PQ}\times\vec{PR}=[183,-55,-108]$ is a normal vector, so $183(x-2)-55(y-9)-108(z+8)=0$

   (b) Find parametric equations for the line through S that is orthogonal (normal) to that plane.
   Answer: $x=5+183t, y=10-55t, z=15-108t$

   (c) Find the area of $\triangle PQR$.
   Answer: $\sqrt{481782}\approx 109.75$

   (d) Find the volume of the parallelpiped formed by P,Q,R,S.
   Answer: dot with $\vec{PS}$ to get 1990

5. (5 pts) Let $f(x,y)=(1+.16x^2y)^3$. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ as functions of x and y.
   Answer: $\frac{\partial f}{\partial x}=3(1+.16x^2y)^2(.32xy)$ and $\frac{\partial f}{\partial x}=3(1+.16x^2y)^2(.16x^2)$

6. (10 pts) Consider a multivariable function with $f(15,8)=47$ and $\nabla f(15,8)=\begin{bmatrix}0.34\\0.85\end{bmatrix}$.

   (a) Find the equation of the tangent plane at the given point.
   Answer: $z=47+.34(x-15)+.85(y-8)$

   (b) Use the linearization to estimate $f(15.25,7.93)$
   Answer: $47+.34(.25)+.85(-.07)=47.0255$

   (c) Let $\vec{v}=[a;7]$. If $a=4$, find $D_{\vec{v}}f(15,8)$.
   Answer: $[.34,.85]\cdot[4,7]/\sqrt{65}\approx .907$

(d) Find the value of a such that $D_{\vec{v}} f(15, 8) = 0$
Answer: solve $.34a + (7)(.85) = 0$ to get $a = -17.5$

7. (4 pts) Suppose $\vec{a}$ and $\vec{b}$ are vectors with $\vec{a} \cdot \vec{b} = 3$ and $\|\vec{a} \times \vec{b}\| = 7$. Find the acute angle between the vectors (in radians).
Answer: $a \cdot b = \|a\|\|b\| \cos(\theta)$ and $\|a \times b\| = \|a\|\|b\| \sin(\theta)$, so $\frac{\sin\theta}{\cos\theta} = \frac{\|a\times b\|}{a \cdot b} = 7/3$, and $\theta = \tan^{-1}(7/3) = 1.166$ (see book 1.4 #2)

8. (4 pts) Consider the sphere $(x - 12)^2 + (y - 7)^2 + z^2 = 100$.
The line $\vec{r}(t) = \begin{bmatrix} 14 + 3t \\ 7 - 8t \\ a + 5t \end{bmatrix}$ is tangent to the sphere. Set up an equation you would solve to find the possible values of a. Do not actually solve it.

9. (7 pts) Consider the line $\vec{r}(t) = \begin{bmatrix} 14 + 3t \\ 7 - 8t \\ 1 + 5t \end{bmatrix}$ and the plane $z = 42 + 0.31(x - 9) + 0.58(y - 17)$.

(a) Find the point of intersection.
Answer: solve $(1 + 5t) = 42 + .31(5 + 3t) + .58(-10 - 8t)$ to get $t = 4.22$; plug-in to get the point $(26.66, -26.76, 22.1)$

(b) Find the acute angle of intersection. (in radians)
Answer: angle between $[3, -8, 5]$ and $[.31, .58, -1]$ is $|\cos^{-1}(-.735) - \frac{\pi}{2}| = .826$ radians