NAME _____

MATH 211 Test 2, Fall 2019

Directions:

- Do not use any notes, books, the internet, or other sources of information.
- You may use a calculator for arithmetic calculations.
- You have 55 minutes. You must work alone; do not communicate with any other person.
- To receive full credit, you must **show all relevant work to completely justify your answer (on separate paper)**.
- 106 points possible, graded out of 100 points.

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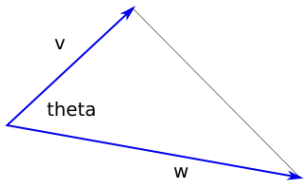
1. (8 pts) Find a vector with length 8 that points in the opposite direction of $\vec{v} = \begin{bmatrix} 2 \\ -6 \\ 9 \end{bmatrix}$.

   Answer: $\frac{-8}{11}\vec{v}$

2. (12 pts) In this picture, the area of the triangle is 50, and $\vec{v} \cdot \vec{w} = 20$. Find the angle θ.



   Answer: $\tan\theta = 100/20$, so $\theta = 78.7°$

3. (12 pts) Consider the points $P(0,0,0)$, $Q(1,2,5)$, $R(3,8,12)$ and $S(7,10,z)$. Find two values of z such that the parallelpiped determined by $\vec{PQ}, \vec{PR}$, and $\vec{PS}$ has volume 50.
   Answer: solve $|(7,10,z) \cdot ((1,2,5) \times (3,8,12))| = |(7,10,z) \cdot (-16,3,2)| = 50$ to get $z = 16, 66$

4. (20 pts) The plane, $z = 7x - 2y - 38$, is intersected by the line $\ell(t) = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \\ 8 \end{bmatrix} t$.

   (a) Find the point of intersection.
   Answer: solve $4 + 8t = 7(1 + 3t) - 2(2 + 5t) - 38$ to get $t = 13$, so $(40, 67, 108)$

   (b) Find the acute angle of intersection (in degrees).
   Answer: angle between $< 3, 5, 8 >$ and $< 7, -2, -1 >$ is $\cos^{-1}(3/\sqrt{5292}) = 87.6365°$, so the angle of intersection is $2.3635°$

5. (36 pts) Let $P(3,8,5)$ and $Q(7,-2,13)$ be two points with $\vec{PQ}$ forming the diameter of a sphere.

   (a) Find the center of the sphere.
   Answer: $(5,3,9)$

(b) Find the equation of the sphere.
Answer: $(x-5)^2+(y-3)^2+(z-9)^2=45$

(c) Which axis (x, y, or z) does the sphere intersect? Find those intercepts.
Answer: intersects the z-axis at $(0,0,9\pm\sqrt{11})$

(d) Is the point $(1,6,5)$ inside or outside the sphere ?
Answer: $4^2+3^2+4^2=41<45$, so inside

(e) There is a polar axis in the direction of $\vec{v}=\begin{bmatrix}7\\8\\9\end{bmatrix}$. Find the equation of the line that passes through the center of the sphere along that axis.
Answer: $\ell(t)=\begin{bmatrix}5\\3\\9\end{bmatrix}+\begin{bmatrix}7\\8\\9\end{bmatrix}t$

(f) Find the equation of the plane that is $\perp$ to the polar axis, splitting the sphere into two hemispheres.
Answer: $7(x-5)+8(y-3)+9(z-9)=0$

6. (18 pts in-class; 12 pts if take-home)
Consider the line $y=mx$ and the parabola $y=x^2$ for $x>0$.

(a) Sketch the graphs, showing the intersection at (m,m).

(b) Set up an equation to find the value of m so that the angle of intersection is exactly $\frac{\pi}{3}$.
Answer: At $x=m$, the slopes of the curves are m and $2m$, represented by vectors $\begin{bmatrix}1\\m\end{bmatrix}$ and $\begin{bmatrix}1\\2m\end{bmatrix}$. Solve $\frac{1+2m^2}{\sqrt{1+m^2}\sqrt{1+4m^2}}=\cos(\frac{\pi}{3})$

(c) Solve the equation for m.
Answer: Use QF to get solve for m^2, but there is no real solution. The angle of intersection never gets as big as $\frac{\pi}{3}$ (my fault!). Interesting question: what is the value of m that yields the biggest angle (minimizes the cosine) ?