NAME ____________________________

MATH 211 Test 3, Fall 2019

Directions:

- Do not use any notes, books, the internet, or other sources of information.
- You may use a calculator for arithmetic calculations.
- You have 55 minutes. You must work alone; do not communicate with any other person.
- To receive full credit, you must **show all relevant work to completely justify your answer (on separate paper)**.
- 105 points possible, graded out of 100 points.

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1. (18 pts) Let $f(x,y) = \frac{8y}{x^2+1}$.

   (a) Find f_x.
   Answer: $\frac{-16xy}{(x^2+1)^2}$

   (b) Find f_y.
   Answer: $\frac{8}{x^2+1}$

   (c) For what value of b does the contour that passes through $(3,b)$ also pass through $(2,10)$?
   Answer: $f(2,10) = 16$, then solve $f(3,b) = 16$ to get $b = 20$

2. (15 pts) Let $f(x,y) = \sin(x^2y\pi) + 5x$.

   (a) Find $\nabla f(x,y)$.
   Answer: $\begin{bmatrix} 2\pi xy\cos(x^2y\pi) + 5 \\ \pi x^2\cos(x^2y\pi) \end{bmatrix}$

   (b) Find the equation of the normal line to the surface of f at the point where $x = 3$ and $y = 7$.
   Answer: $\ell(t) = \begin{bmatrix} 3 \\ 7 \\ 15 \end{bmatrix} + \begin{bmatrix} -42\pi + 5 \\ -9\pi + 5 \\ -1 \end{bmatrix} t$

3. (12 pts) Let $z = f(x,y) = 2^{(x^2+y^2)}$.

   (a) Find the range of this function.
   Answer: $x^2 + y^2 \geq 0$, so $z \geq 1$ and the range is $[1, \infty)$

   (b) Find the radius of the contour corresponding to $z = 64$.
   Answer: $x^2 + y^2 = \log_2(64) = 6$, so the radius is $\sqrt{6}$

4. (13 pts) In the physics lab there is a mysterious contraption with two dials labeled x and y, and a voltage display that currently reads $V = 380$. Out of curiosity you:

   - turn the x dial up 5 clicks, and the voltage increases to 410
   - then, without turning the x dial back, you turn the y dial up 2 clicks and V drops down to 400

   (a) Estimate ∇V at the dial settings after you turned x, but before you turned y.
   Answer: using difference quotients, $V_x \approx 30/5 = 6$ and $V_y \approx -10/2 = -5$

   (b) Find dV if $dx = -3$ and $dy = 4$.
   Answer: $dV = (6)(-3) + (-5)(4) = -38$

5. (26 pts) Suppose $f(23,52) = 17$ and $\nabla f(23,52) = \begin{bmatrix} 0.82 \\ 0.27 \end{bmatrix}$.

   (a) Write the equation of the tangent plane at that point.
   Answer: $z = 17 + .82(x-23) + .27(y-52)$

   (b) Estimate $f(23.37, 51.48)$.
   Answer: $17 + .82(0.37) + .27(-.52) = 17.163$

   (c) Find the slope of the surface in the direction 30° west of north.
   Answer: $.82\cos(2\pi/3) + .27\sin(2\pi/3) = -.176$

   (d) At the given point, the vector $\vec{v} = \begin{bmatrix} 4 \\ b \end{bmatrix}$ points along a contour. Find the value of b.
   Answer: set $\vec{v} \cdot \nabla f = 0$, so $4(.82) + .27b = 0$, implies $b = -12.15$

6. (14 pts) You are standing on the surface of a rolling hill, and the differential of your elevation is $dz = 0.15dx - 0.23dy$.

   (a) Find the slope of the hill in the direction $\vec{v} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$.
   Answer: $\frac{(5)(.15)+(-2)(-.23)}{\sqrt{5^2+2^2}} = .2247$

   (b) If you plant a flag that points straight up, find the angle it makes with the ground.
   Answer: the normal vector is $\begin{bmatrix} .15 \\ -.23 \\ -1 \end{bmatrix}$; the flag is $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, and the angle between those is $\cos^{-1}(-1/\sqrt{1.0754}) = 164.65°$, so the acute angle with the ground is $|90 - 164.65| = 74.65°$
   Another method: the steepest slope is $\|\nabla f\|$, and the vertical flag makes an angle of $90 - \tan^{-1}(\|\nabla f\|)$.

7. (7 pts) If $\nabla f = \begin{bmatrix} 10 \\ b \end{bmatrix}$, find the value of $b > 0$ such that the steepest ascent direction has slope 12.
   Answer: $\|\nabla f\| = \sqrt{100 + b^2} = 12$ implies $b = \sqrt{44}$