NAME ____________________________

MATH 211 Test 5, Fall 2019

Directions:

- This exam is open book/notes. But do not use the internet.
- You may use a calculator for arithmetic calculations.
- You have 55 minutes. You must work alone; do not communicate with any other person.
- To receive full credit, you must **show all relevant work to completely justify your answer (on separate paper)**.
- 105 points possible, graded out of 100 points.

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1. (21 pts) Let R be bounded by $x = 0$, $y = x$, and $y = \frac{6}{x+1}$. Suppose that $\iint\limits_R f(x,y)dA = 75$.

   Find the average value of f on R.
   Answer: $\iint_R 1dA = \int_0^2 6(x+1)^{-1} - xdx = 6\ln(3) - 2$, so the average value of f is $\frac{75}{6\ln(3)-2} \approx 16.33$

2. (21 pts) Evaluate the integral by switching the order of integration.

$$\int_0^1 \int_{3x}^3 y^4 \cos(xy^2)dydx$$

   Answer: draw a picture, $\int_0^3 \int_0^{y/3} y^4 \cos(xy^2)dxdy = \int_0^3 y^2 \sin(y^3/3)dy = 1 - \cos(9) \approx 1.91$

3. (21 pts) Evaluate this improper integral:

$$\int_0^\infty \int_{x^2}^\infty 12xe^{-y}dydx$$

   Answer: $\int_0^\infty 12xe^{-x^2}dx = 6$

4. (21 pts) A room occupies the region R bounded by $|y - 10| = 3$ and $y = |x|$. The height of the ceiling is given by $f(x,y) = 15 - \cos(xy/9)$, and the floor is the xy-plane.

   (a) Set up (but do not evaluate) a double integral to find the volume of the room.
   Answer: $\int_7^{13} \int_{-y}^y f(x,y)dxdy$

   (b) Set up (but do not evaluate) a calculation to find the average distance from points in R (on the room's floor) to the origin.
   Answer: the room's area is 120, so $\frac{1}{120}\int_7^{13}\int_{-y}^y \sqrt{x^2+y^2}dxdy$

5. (21 pts) For $a > 0$, define

$$f(a) = \int_0^{a/2} \int_0^a (x+y)^2 dydx$$

   (a) Work out a simple formula for $f(a)$, and then solve $f(a) = 1000$.
   Answer: $\frac{1}{3}\int_0^{a/2}(x+a)^3 - x^3dx = \frac{1}{12}(3a/2)^4 - (a/2)^4 - a^4 = \frac{a^4}{3} = 1000$, so $a \approx 7.4$

   (b) Find the value of a at which $f'(a) = \frac{df}{da} = 1000$.
   Answer: solve $4/3a^3 = 1000$ to get $a \approx 9.09$